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METHODS OF MODULATION

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B. H. Andrews

METHODS OF MODULATION

by

Burton Howell Andrews
Lieutenant Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE

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the thesis requirements for the degree of

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in

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School.

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PREFACE

This work was undertaken at the United States Naval Postgraduate School during the writer's third year at that Institution and as a result of his interest in the relative merits of the many possible methods of impressing intelligence information upon a radio carrier wave.

The writer extends his appreciation to the various commercial radio organizations, especially the Federal Telecommunications Laboratories, and to Mr. H. E. Fowler, Librarian at this School, for their efforts in procuring and making available requested technical literature of much value in this work.

TABLE OF CONTENTS

CHAPTER I. EXISTING AND POSSIBLE METHODS.
1. Introduction.
2. Methods.

CHAPTER II. PERFORMANCE CRITERIA.
1. Theoretical relationships.
2. Practical analysis.

CHAPTER III. BASIC SYSTEMS.
1. Amplitude Modulation (AM).
2. Frequency modulation (FM).
3. Phase modulation (PM).

CHAPTER IV. TIME DIVISION SYSTEMS.
1. Pulse amplitude plus amplitude modulation (PAM-AM)
2. Pulse amplitude plus frequency modulation (PAM-FM)
3. Pulse phase plus amplitude modulation (PPM-AM)
4. Pulse phase plus frequency modulation (PPM-FM)
5. Pulse width plus amplitude modulation (PWM-AM)
6. Pulse width plus frequency modulation (PWM-FM)
7. Pulse number and pulse code modulation (PNM,PCM)

CHAPTER V. FREQUENCY DIVISION SYSTEMS.
1. Amplitude plus amplitude modulation (AM-AM)
2. Amplitude plus phase modulation (AM-PM)
3. Single side-band plus amplitude modulation (SS-AM)
4. Single side-band plus phase modulation (SS-PM)
5. Frequency plus amplitude modulation (FM-AM)
6. Frequency plus frequency modulation (FM-FM)

CHAPTER VI. DISTORTION.
1. Introduction.
2. Frequency Division Systems.
3. Time Division Systems.

CHAPTER VII. CONCLUSIONS.

CHAPTER VIII. PCM SYSTEMS.
1. Principles of operation.
2. Coding.
3. Experimental system performance.

LIST OF ILLUSTRATIONS

		Page
Figure 1.	Frequency Division Systems (A) Time Division Systems (B)	7
Figure 2.	Noise Impulse.	13
Figure 3.	Frequency Modulation Noise Spectrum.	26
Figure 4.	Frequency Modulation Signal Spectrum.	26
Figure 5.	(a) Minimum length pulses, PAM-AM. (b) Maximum length pulses, PAM-PM.	35
Figure 6.	Effect of noise on PPM pulse.	39
Figure 7.	Effect of noise on PWM pulse.	44
Figure 8.	Effect of signal on PWM pulse.	45
Figure 9.	Possible groups for three-digit code.	49
Table 1.	System performance under specified conditions.	59
Figure 10.	(a) Equal level quantizing characteristic. (b) Tapered quantizing characteristic.	70
Figure 11.	Schematic of Coding Tube Circuit.	73
Figure 12.	Three digit coding, aperture plate and output signal.	74
Figure 13.	Noise levels in experimental system.	78

TABLE OF SYMBOLS AND ABBREVIATIONS

a	Nominal pulse duration; the time during which the pulse exceeds 50% amplitude.
Δa	Change in pulse duration. Parenthesis and subscript indicate cause of change.
B	The RF or IF bandwidth, as applicable. Equal to $(\frac{1}{\tau})$ for minimum length pulses.
B_{min}	The minimum value of B for system operation.
B_t	The required bandwidth at threshold operation.
$\frac{C}{N}$	The ratio of peak carrier voltage to peak noise voltage after selection and before detection.
CREST FACTOR:	The ratio of peak volts to rms volts, with respect to random noise voltages.
db	Decibel(s)
cps	Cycles per second.
D	Channel pulse sub-carrier duty factor, the product of pulse duration and channel pulse repetition frequency. $D = a\, f_p$.
D_c	Duty factor of a pulse modulated carrier wave.
D_i	Distortion; given in percent.
D_o	Modulating signal duty factor. $D_o = nD$.
ΔD	Change in channel pulse duty factor. Parenthesis and subscript indicate cause of change.
e	Instantaneous voltage.
e_c	Instantaneous carrier voltage.
e_n	Instantaneous noise voltage.
e_r	Instantaneous resultant voltage, the vector sum of e_c and e_n.
e_s	Instantaneous signal voltage.
e_{no}	Instantaneous noise output voltage.

e_{so} Instantaneous signal output voltage.

E_m Peak value of complex modulating signal.

E_n Peak value of nth subchannel forming the complex modulating signal.

f Instantaneous frequency.

F_a The highest intelligence frequency of any channel.

f_c Carrier frequency.

F_d Maximum frequency deviation of carrier from its unmodulated frequency f_c.

f_{dn} The peak deviation of the nth subcarrier caused by the intelligence impressed upon it.

F_h The highest frequency component of the complex modulating signal; the highest frequency recovered at the output of a group detector.

f_n Noise frequency.

f_p Channel pulse repetition frequency.

f_r Resultant frequency; $f_r = (f_c - f_n)$.

f_s Signal frequency.

f_w Frequency spacing between subcarriers of a frequency division system.

G The Improvement; the ratio of signal-to-noise ratios, comparing a single channel of the system under consideration to a single channel AM system using the same carrier power.

IF Intermediate frequency.

K Boltzmann's Constant, $= 1.37 \times 10^{-23}$ joules per degree Kelvin.

k An arbitrary constant.

k_1 The ratio between f_p and F_a.

M The Merit Factor; the reciprocal of the product of required bandwidth and required received power.

mc Megacycles.

M_c The amount any one subchannel can be allowed to modulate the carrier, compared to 100%.

n The number of channels in a multichannel system.

θ_m Maximum equivalent phase displacement due to signal modulation in a PPM system.

θ_n Maximum equivalent phase displacement due to noise modulation in a PPM system.

P_c Average received power. The carrier power of a single channel AM system.

P_n Average noise power.

p_r Pulse repetition frequency of a complex modulating signal in a time division system. $p_r = n f_p$.

P_s Average signal output power.

P_t Required received average power at threshold.

p The number of pulse combinations or quantum levels of the intelligence signal in a PNM system.

$\frac{P_c}{P_n}$ The ratio of average carrier power to average noise power. For a noise crest factor equal to four and a carrier duty factor D_c , the ratio is equal to $8D_c\left(\frac{C}{N}\right)^2$

$\frac{P_s}{P_n}$ The ratio of average signal power to average noise power. Equals the square of the signal-to-noise ratio.

PI The Power Improvement; the ratio of the minimum required power for a single channel AM system to the minimum required power for the system under consideration, for the same operating conditions. Expressed in db.

Q A specified desired ratio of P_s to P_n .

rms Root-mean-square.

RF Radio Frequency.

$\frac{S}{N}$ The ratio of rms signal voltage to rms noise voltage.

T Temperature in absolute degrees, Approx. 293^o

THRESHOLD: The condition of unity for the $\frac{C}{N}$ ratio.

τ The duration of a minimum length pulse; pulse build-up and decay time, the value $\frac{1}{B}$.

t Time

w Angular velocity.

w_c Carrier angular velocity.

w_n Noise angular velocity.

w_r Resultant angular velocity. $w_r = (w_c - w_n)$.

w_s Signal angular velocity.

CHAPTER I

EXISTING AND POSSIBLE METHODS

1. Introduction.

The unprecedented demands experienced during the past decade for allocations in the radio frequency spectrum have produced energetic research in two general directions:

1. The extension of the usable portion of the radio frequency spectrum to higher frequencies.

2. Means of realizing more efficient utilization of the presently available spectrum space.

This paper is concerned with the second of these objectives and primarily with its theoretical aspects. The designer of commercial equipment and communications systems is inevitably surrounded by specifications which necessitate departures from the theoretical ideal. Space, weight, power, range, cost, degree of complexity tolerable, acceptable distortion and signal-to-noise ratios, and the form of the original information are but a few of these factors. Only those directly affecting the theory involved will be considered herein. Of the many possible systems, several of the apparently most practicable are analyzed in some detail in terms of certain performance criteria involving the basic systems characteristics. The final sections of the paper are devoted to the

relatively new Pulse Code Modulation system which though highly advantageous from a theoretical point of view has, until quite recently, been almost wholly impracticable because of the complex circuitry involved.

2. Methods.

A definition of modulation is "A process by which some characteristic of a carrier is varied with time in accordance with a modulating wave." (IRE Committee on Modulation Standards). One notes here that the definition implies no restrictions upon the nature of the carrier wave or of the modulating wave. For instance, the carrier may be a continuous wave, a direct current, a pulsed continuous wave, etc.; the modulating wave may be any of these, or it may itself be a modulated carrier (in which case it is referred to as a sub-carrier). It is apparent that there are many possible combinations which might conceivably be used. Some of these combinations will now be classified, and the more important ones will be analyzed in greater detail later. Direct current carrier systems will not be discussed as they embrace a field outside the interest of this paper. Systems employing pulsed continuous waves as the carrier may find use; however, they are inferior to systems employing continuous wave carriers with similar modulation unless the pulse rate is very high and the transmitted bandwidth very great, and hence will not be taken up in detail.

Three basic methods for modulation of a continuous wave carrier have been developed to a high degree:

AMPLITUDE MODULATION

a) Double side band plus carrier (AM)

b) Single side band and partially or wholly suppressed carrier (SS)

FREQUENCY MODULATION (FM)

PHASE MODULATION (PM)

Under certain restricted conditions, simultaneous modulation of one carrier wave by more than one method can be used; however, such operation involves a decrease in the obtainable S/N ratio with either method alone, and/or increased difficulties with interchannel cross-modulation products. These methods are therefore not considered a practical solution to the general problem of spectrum space conservation.

The most radical changes have come about in the character of the modulating wave. The transmission of a relatively narrow band of audio frequencies requires inherently but a small fraction of the available bandwidths of high frequency equipment, because a given amount of information per unit time in a given form requires a bandwidth which is independent of the ultimate frequency of transmission and the available bandwidth at the transmission frequency is essentially a percentage function of that frequency. Obviously, the extra bandwidths available might advantageously be used to provide greater fidelity of transmission, better S/N ratios, the transmission of more information per unit time through the same equipment, or combinations of these factors. The

development of modern multi-plexing techniques was one logical result.

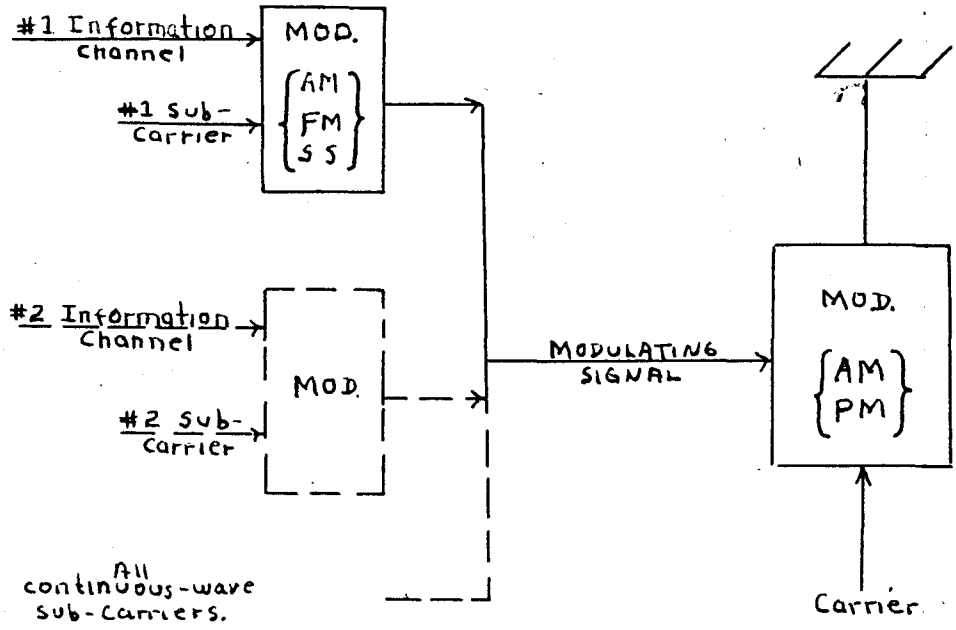
All multiplexing systems have the common characteristic that the carrier wave is modulated by a modulating wave which contains information from more than one information channel. Basically, each information channel is furnished a subcarrier which is modulated by that channel; the modulated subcarriers are then combined and used to modulate a carrier (a double modulation system) or another subcarrier (a triple modulation system) and so on to any desired degree of complexity. At the present state of the art any gains which, if they exist, might be obtained with systems using more than triple modulation are more than offset by the complexities of equipment design, construction, and adjustment, and the difficulties of keeping interchannel cross-modulation within acceptable limits. It has been shown, in fact, that all triple modulation systems are inferior to the related double modulation systems (Landon (9)).

The use of pulsed continuous waves as subcarriers or intermediate subcarriers offers no advantages and presents the same disadvantages mentioned for their use as carriers. Continuous waves may be, and are, used in the following manner: A group of modulated continuous wave subcarriers is used as a modulating signal for a

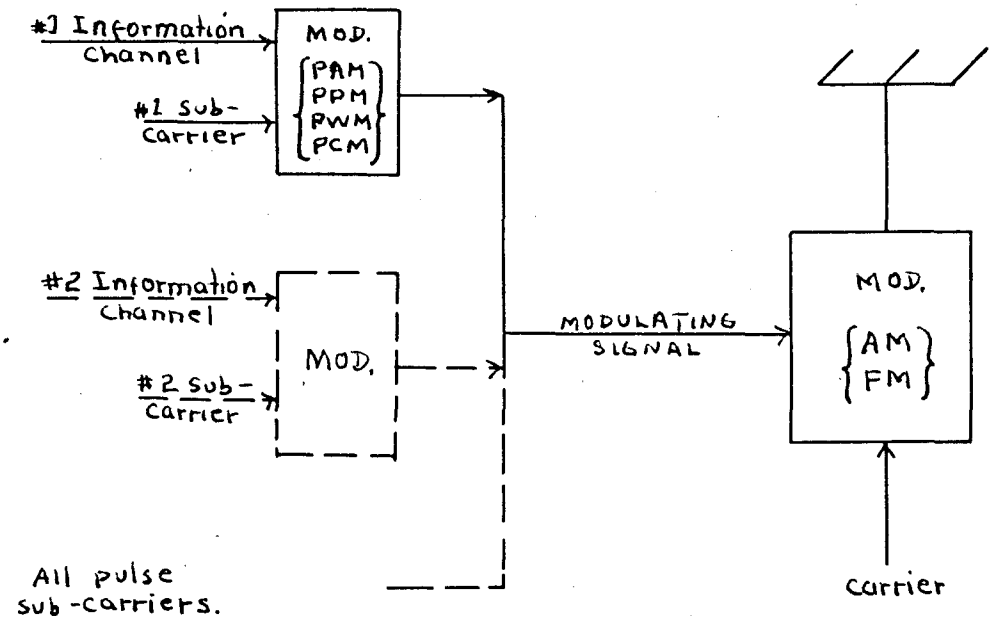
higher frequency carrier; the frequency separation between subcarriers will be dictated by the type of modulation imposed upon them. This is a double modulation system. A group of double modulation systems with spaced (sub) carriers may be used to modulate a still higher carrier frequency, giving a triple modulation system. Systems of this type are called frequency division systems.

A train of d.c. pulses may be used as a subcarrier; systems of this type are called time division systems. Each subcarrier has the same frequency, but the pulses are of such lengths and so phased that when all the modulated subcarriers are combined, the pulses interleave but do not overlap. The group of subcarriers is then used to modulate a carrier or intermediate subcarrier as before described. The pulse subcarriers may be modulated by the information channel in a wide variety of ways, involving variation in pulse position, pulse shape, or the number and/or sequence of a series of pulses. The pulse repetition frequency will be dictated by the type of modulation imposed upon the pulse-train subcarrier.

The possible double-modulation combinations are indicated in Fig.1, where the appropriate choice of FM or PM is shown in cases to which either might apply. Methods of pulse modulation apparently not included will be found so similar to one of the listed methods as not



A. Frequency Division Systems.



B. Time Division Systems.

Figure 1.

to warrant separate listing; they will be discussed later in conjunction with the related method listed.

The symbols used are as follows:

AM : Amplitude modulation, carrier plus two sidebands.

FM : Frequency Modulation.

PM : Phase Modulation.

SS : Amplitude Modulation, Single sideband, suppressed carrier.

PAM : Pulse Amplitude Modulation.

PPM : Pulse Phase Modulation.

PWM : Pulse Width Modulation.

PNM : Pulse Numbers Modulation.

PCM : Pulse Code Modulation.

In the following work, a designation such as PAM-AM is used to signify that a group of Pulse Amplitude Modulated subcarriers is used as a modulating signal to Amplitude Modulate a carrier, etc.

CHAPTER II

PERFORMANCE CRITERIA

1. Theoretical relationships.

System performance will be analyzed and compared on the basis of the following criteria:

a. Distortion, including interchannel modulation.

b. Relative signal-to-noise ratios.

c. Required transmission bandwidth.

d. Required received power.

The relation among signal-to-noise ratio, required transmission bandwidth, and information capacity has long been obscure. It was readily deducible and often observed that an increase in the rate of information transmission required an increase in bandwidth utilized: the accompanying change, if any, in signal-to-noise ratio was not precisely defined. In 1935 Armstrong (1) proved to skeptical engineers that better signal-to-noise ratios could be obtained by increasing the bandwidth, for a given rate of information transmission.

The true mathematical relation among these factors has but recently been announced [Shannon (19)] . This relation takes the relatively simple form

$$C = kBZ$$ where

C is a measure of information transmission rate.

B is the bandwidth of transmission.

Z is the number of db expressing $\left(\frac{P_s}{P_n} + 1\right)$

The implications of this expression are great. For instance, examine one of the better systems in present use: FM. Doubling the bandwidth betters the signal-to-noise ratio by 6db; quadrupling betters it by 12db. The improvement is additive. But Shannon's expression indicates that for an ideal system the improvement in signal-to-noise ratio should be multiplicative rather than merely additive. (Using the same figures, the improvement would be 36db for quadrupled bandwidth). All presently known modulation systems fall far short of the indicated ideal. The prime problem of future communications systems research may well be the development of systems capable of realizing the performance predicted by the theoretical relationship given above.

2. Practical Analysis.

The formulation of exact analytical expressions completely defining the performance of a complex communication system is in general either impossible or productive of results which are so complicated as to be of little or no practical value. However, the use of justifiable approximations and logical reasoning can produce in most cases quantitative results sufficiently accurate for engineering purposes. This is particularly true in the evaluation of the effects of noise upon a system; its very nature precludes the possibility of writing down an exact expression describing it fully, but under certain restrictive conditions (commonly met in practice) the noise can be synthesized and its effects quite accurately predicted.

Two types of noise are encountered, qualitatively defined as follows:

IMPULSE NOISE: Impulse noise is noise characterized by transient disturbances separated in time by quiescent intervals.

Some care must be taken in the application of this definition. If the bandwidth of a system is such that a series of noise impulses is faithfully passed, a rather considerable amount of such noise is tolerable because of its short duration. However, the bandwidth of the system

may be reduced to the point where the wave trains produced by successive pulses overlap. The effects in this case will be considerably worse than those predicted on the basis of individual transient pulses. Where the term impulse noise is used in this report, it will be understood that the interval between successive pulses is such that overlapping does not occur. Under these conditions, the peak noise voltage (due to impulse noise) within a system will be directly proportional to the system bandwidth, as shown by the following:

Consider a noise impulse of build-up time τ and mean duration between half-amplitude points a and amplitude E (See Figure 2). This pulse can be represented by a pair of Fourier Integrals as follows:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(w)\, e^{jwt}dt$$

$$F(w) = \int_{-\infty}^{+\infty} f(t)\, e^{-jwt}dw = \frac{2E}{w}\,\frac{\sin(\frac{w\tau}{2})}{(\frac{w\tau}{2})}\,\sin(\frac{w\,a}{2})$$

The plot of F(w) may be interpreted as a plot of the amplitudes of the components of f(t) as a function of w. At the operating frequencies where impulse noise is objectionable, it can be assumed that

1) $\tau << \frac{1}{f}$ and

2) $(f_2 - f_1) << f$ and hence over the bandwidth involved $(f_2 - f_1)$ the components are of essentially equal amplitude and in phase at the beginning and end of

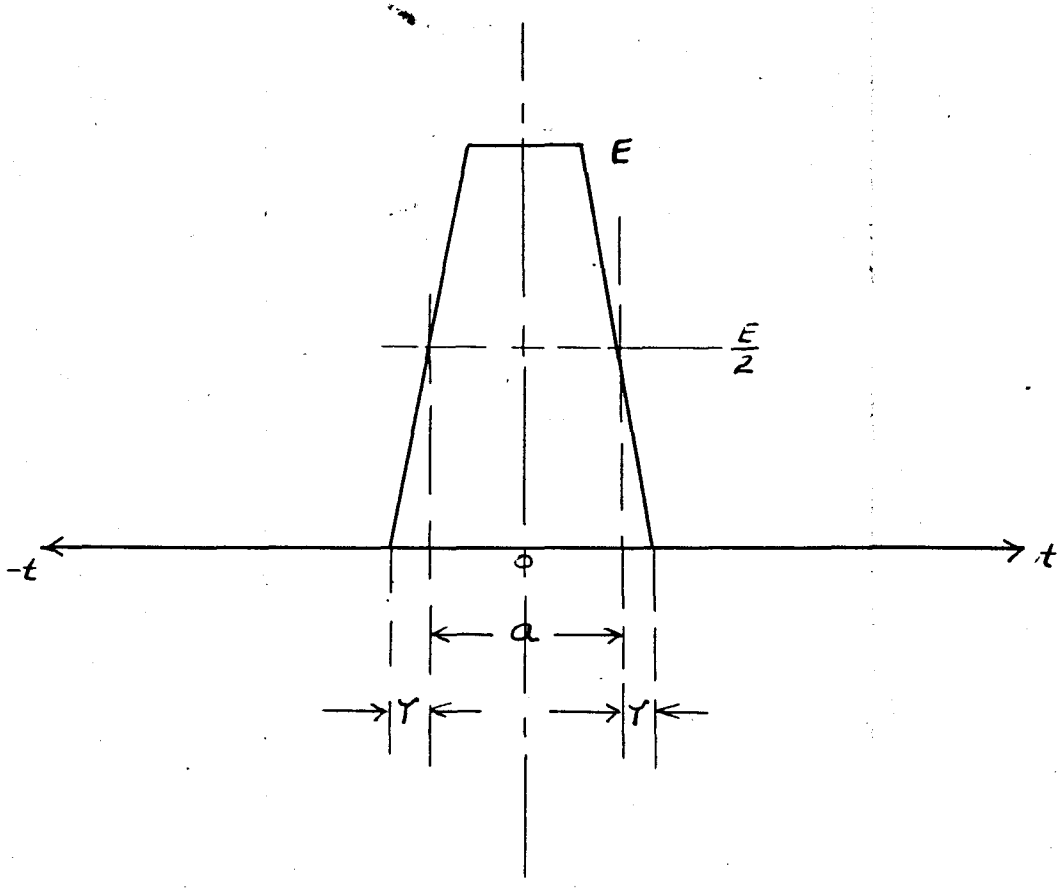


Figure 2.

the pulse. Under these conditions they will add linearly and the peak value will therefore be directly proportional to bandwidth. It is further obvious that impulse noise becomes less and less of a problem as we progress to higher and higher frequencies. Over the bulk of the frequency range to which multi-channel communications is applicable (from about 50mc to beyond 30,000mc) impulse noise is of minor importance compared to the second type, now taken up:

RANDOM NOISE: Random noise is noise characterized by a large number of overlapping transient disturbances occurring at random. (Fluctuation noise and random noise are terms often used synonomously).

Within this definition is included the familiar thermal-agitation noise inherent in any dissipative medium. This type of noise represents an energy density distributed essentially uniformly over the entire frequency spectrum; ie, the random noise power within a system will be proportional to the bandwidth involved. The rms random voltage then will be proportional to the square-root of bandwidth. A great amount of theoretical and experimental work has been done* on the subject of random noise: the most important conclusions for the purpose of this paper are the following: (the term noise voltage will hereafter refer to random noise unless stated otherwise)

---

* Johnson (4),Williams (7), Nyquist (8),Lardon (13) Carson (15),Rice (21)

1. Noise voltage is proportional to (bandwidth)$^{\frac{1}{2}}$

2. Peak noise voltage amplitude is approximately four times rms value (crest factor = 4)

3. The noise voltage existing in a circuit which is frequency selective can be expressed as

$e_n = f(t)\mathrm{Sin}\left[wt + \phi(t)\right]$ after passing through the selective network. Here, f(t) and $\phi(t)$ are functions of time and $\neq 0$, and w is an arbitrary constant such that $w_1 \leq w \leq w_2$ and w_1 and w_2 are the angular frequency limits of the bandwidth passed, Furthermore, if $\frac{w_2 - w_1}{w} << 1$, then $\frac{df(t)}{dt} << 1$ and $\frac{d\phi(t)}{dt} << 1$. The importance of this lies in the fact that it allows us, in analyzing the effect of noise upon a system meeting the above requirements (and they almost invariably do), to consider the effect of one noise component of fixed amplitude and frequency, and then by graphical or mathematical integration to sum up the total effect as the noise frequency varies over the entire bandwidth.

Using these principles, the S/N ratio in one output channel of a communications system can be computed in terms of various system parameters. For convenience in comparison, a factor G, the Improvement, will be derived; it is the ratio of Signal-to-Noise ratios, comparing a single channel of the system under consideration to a single channel AM system

having the same carrier power. To facilitate comparisons and computations, certain operating conditions will be stipulated:

1) The modulating signal supplied to the transmitter shall be of high quality (noise in each channel at least 60db below signal)

2) Operation is well above the threshold level.

3) The pulse (time division) systems will employ straight-sided pulses with the time of rise designated as τ ; the necessary pass-band for such a pulse will be taken* as $B = \frac{1}{\tau}$. The minimum length of pulse that a bandwidth B will pass is a triangular pulse with equal times of rise and decay, equal to τ ; in this case, pulse duration is also τ , since pulse duration is defined as the time during which the pulse exceeds 50% amplitude. The factors Bandwidth, pulse duration, and pulse frequency enter heavily in the determination of inter-channel cross-modulation.

The improvement factor G does not, in itself, give a complete comparison, since it does not specify the relations among threshold levels for the various systems. It will be desirable to compute the necessary received power to realize a specified S/N ratio in one output channel. This power can be computed in terms of G and the desired S/N ratio as follows:

---

* Guilleman(6) Vol.II Chapt. XI

It will be shown that for an AM single channel system, $\left(\frac{S}{N}\right)_{AM} = \frac{C}{N}\sqrt{\frac{B}{2F_a}}$ from which

$$\left(\frac{S}{N}\right)^2_{AM} = \left(\frac{P_s}{P_n}\right)_{AM} = \frac{B}{2F_a}\left(\frac{C}{N}\right)^2$$

and for a noise crest-factor equal to 4, this becomes

$$\left(\frac{P_s}{P_n}\right)_{AM} = \frac{B}{2F_a}\,\frac{1}{8}\,\frac{P_c}{P_n}$$

In a properly operated AM system, $B = 2F_a$; hence,

$$\left(\frac{P_s}{P_n}\right)_{AM} = \frac{1}{8}\,\frac{P_c}{P_n} = \frac{1}{8}\,\frac{P_c}{KTB} = \frac{P_c}{16KTF_a}$$ where

K = Boltzmann's constant 1.37×10^{-23} joules/deg. Kelvin

T = absolute temperature, approximately 293 deg.

For any other system, we have by definition

$$\left(\frac{S}{N}\right)_x = G\left(\frac{S}{N}\right)_{AM}$$ from which

$$\left(\frac{P_s}{P_n}\right)_x = G^2\left(\frac{P_s}{P_n}\right)_{AM} = \frac{G^2 P_c}{16KTF_a}$$

and solving for P_c we obtain

$$P_c = \frac{16KTF_a}{G^2}\left(\frac{P_s}{P_n}\right)_x$$

where P_c is required received power to realize the specified signal-to-noise ratio $\left(\frac{S}{N}\right)_x = \left(\frac{P_s}{P_n}\right)_x$

For many systems, G is a function of B, and therefore P_c is a function of the reciprocal of B in these systems. In these systems, P_c will be the minimum possible when the bandwidth is made sufficiently large to produce threshold operating conditions, assuming that the improvement factor can be maintained down to this point. This assumption is open to question, but appears to be justifiable in theory and closely approachable in practise. The bandwidth necessary to produce this condition can be derived from the relation

$$\frac{P_c}{P_n} = 8D_c \left(\frac{C}{N}\right)^2$$

where D_c is the carrier duty factor. At threshold, by definition C/N equals unity; therefore

$$P_t = P_n \, 8D_c = 8 \, KTB \, D_c$$

Equating this to the previously derived expression for P_c and solving for B gives the bandwidth required at threshold for the specified S/N ratio:

$$B_t = \frac{2F_a}{G^2 D_c}\left(\frac{P_s}{P_n}\right)x$$

For some systems, the bandwidth required on this basis is so great as to be wholly impracticable; in such cases, the maximum allowable bandwidth is arbitrarily chosen and the

power required calculated on that choice. For systems in which the required power is not a function of bandwidth, the minimum bandwidth is determined by cross-modulation considerations.

Two other figures useful for comparison of systems will be calculated. For equal signal-to-noise ratios in the channel outputs of the systems compared, they are defined as follows:

The merit Factor, M, defined as the reciprocal of the product of the minimum required power and the bandwidth.

The Power Improvement, PI, defined as the ratio of the minimum required power for a single channel AM system to the minimum required power for the system considered. This ratio is expressed in db. To afford a numerical comparison for stipulated operation conditions, Table 1 page 59 tabulates the Improvement (G), the Power Improvement (PI), the Merit Factor (M), the required minimum power (P) and the bandwidth (B). These values were computed on the basis of 100 channel operation, intelligence band 0 to 3000 cycles (F_a = 3000) for each channel, sub-carrier separation f_w = 4000 cycles for ss-xx; 7000 cycles for all others, required signal-to-noise ratio 50 db ($Q = 10^5$ on a power basis). In systems for which the threshold bandwidth was greater than 10 mc, the bandwidth was arbitrarily set at 10 mc except for the PCM and PNM systems; in the latter

two the bandwidth was set at the minimum required for operation of the systems under any conditions. The table is made out in descending order of the Merit Factor (M).

CHAPTER III

BASIC SYSTEMS

1. Amplitude Modulation (AM)

Consider a carrier voltage and a noise voltage respectively given by

$$e_c = C\,(1 + \sin w_p t)\sin w_c t \;=\; A(t)\sin\theta$$

$$e_n = N\sin w_n t \;=\; N\sin\phi$$

to exist at the input terminals of the receiver, giving a C/N ratio equal to C/N. With linear conversion, the same C/N will exist at the detector unput, assuming that the R.F. bandwidth equals the I.F. bandwidth. If the latter condition does not obtain, then

$$C/N)_{det} = C/N_{input} \text{ times } \left(\frac{\text{R.F. bandwidth}}{\text{I.F. bandwidth}}\right)^{\frac{1}{2}}$$

The ratio of R.F. bandwidth to I.F. bandwidth being essentially the same for all systems, no loss in generality results if this constant factor is omitted. Unless otherwise stated, C/N ratio will hereafter refer to the detector input.

Vector addition of the carrier and noise voltages produces a resultant voltage given by

$$e_r = \sqrt{[A(t)]^2 + N^2 + 2A(t)N\cos(w_c - w_n)t \sin\psi} \qquad \text{where}$$

$$\psi = \theta + \tan^{-1}\left[\frac{N\sin(\phi - \theta)}{A(t) + N\cos(\phi - \theta)}\right]$$

The detector responds only to variations in the amplitude of the impressed signal. Under the specified operating conditions, $C/N \gg 1$; and eliminating amplitude variations due to signal modulation by setting $w_p = 0$, the expression for the amplitude of e_r reduces to

$$|e_r| = C\left(1 + \frac{2N}{C}\cos w_r t\right)^{-\frac{1}{2}}$$

$$\cong C\left(1 - \frac{N}{C}\cos w_r t\right) \qquad \text{where } w_r = (w_c - w_n)$$

Without noise, the amplitude of the wave applied to the detector is $C(1 + \sin w_p t)$. The effect of the noise is that of a modulating signal with a modulation factor of $\frac{N}{C}$ and an angular frequency of w_r. Hence the signal-to noise ratio at the detector output over the audio frequency range 0 to $\frac{B}{2}$ is equal to the carrier-to-noise ratio at the detector input when the I.F. bandwidth equals $2F_a$. If the I.F. bandwidth is greater than required, we need consider the $\frac{C}{N}$ ratio only within that portion of the IF band $2F_a$ wide since all other noise components are eliminated by the low-pass filter following the detector output circuit.

$$\frac{S}{N} = \frac{C}{N}\left(\frac{B}{2F_a}\right)^{\frac{1}{2}} = \frac{C}{N}$$

This result can be arrived at by the following somewhat simpler line of reasoning. Full amplitude modulation produces two side-bands each of one-half carrier amplitude. In other words, for a given carrier-to-noise ratio over an RF bandwidth B, each side-band has the ratio $\frac{1}{2}\frac{C}{N}$. The detector folds the two side bands upon each other, the signal components adding linearly but the noise components adding in rms fashion; hence over a bandwidth B in the detector output

$$\frac{S}{N} = \frac{C}{N}\frac{1 + 1}{\sqrt{2^2 + 2^2}} = \frac{1}{\sqrt{2}}\frac{C}{N}$$

But the audio output system is low-pass up to F_a; hence over the output band 0 to F_a

$$\frac{S}{N} = \frac{C}{N}\frac{1}{\sqrt{2}}\sqrt{\frac{B}{F_a}} = \frac{C}{N}$$

The expressions for the other performance figures become:

$$G = 1$$

$$B = B_{min} = 2F_a$$

$$P_c = 16\ KTF_a\ Q$$

$$M = \frac{1}{32\ KT\ F_a^2\ Q}$$

$$PI = 0\ db$$

2. Frequency Modulation (FM)

As in amplitude modulation, we again consider the resultant voltage produced by the vector addition of a carrier and a noise voltage, but in this instance the carrier is frequency modulated. We have:*

$$e_c = C\sin(w_c t + \beta\cos w_p t) = C\sin\theta \quad ; \quad (\beta = \frac{F_d}{f_p})$$

$$e_n = N\sin w_n t = N\sin\phi \qquad \text{from which}$$

$$e_r = E_r\sin\psi(t) \qquad \text{where}$$

$$E_r = \sqrt{C^2\sin^2\theta + N^2\sin^2\phi + 2CN\cos(w_r t + \beta\cos w_p t)}$$

$$\text{and } \psi = \theta + \tan^{-1}\left[\frac{N\sin(\phi - \theta)}{C + N\cos(\phi - \theta)}\right] = \theta + \tan^{-1}g(t)$$

The detector is considered non-responsive to amplitude variations, or it is assumed that the use of limiting has removed such variations from the wave applied to the detector; we are interested in the instantaneous frequency of the resultant wave e_r, and especially in the deviations from center frequency caused by the presence of the noise voltage. The instantaneous frequency is given by

$$f = \frac{1}{2\pi}\frac{d\psi}{dt} = \frac{1}{2\pi}\left[\frac{d\theta}{dt} + \frac{\frac{dg(t)}{dt}}{1 + g(t)^2}\right]$$

Carrying out the indicated operations results in an unwieldy expression; however, since again N/C<<1, and eliminating

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* Armstrong (1), Crosby (5), MIT Staff (12)

variations due to modulating signal by setting $\beta = 0$, we have finally

$$f = f_c - (\frac{N}{C})^2 f_r - (\frac{N}{C}) f_r \cos w_r t \qquad \text{where } f_r = f_c - f_n$$

Hence, the maximum deviation due to noise is

$$(\frac{N}{C})^2 f_r + (\frac{N}{C}) f_r = (\frac{N}{C}) f_r (1 + \frac{N}{C}) \cong \frac{N}{C} f_r$$

The output voltage of a linear discriminator is some constant times the instantaneous deviation from center frequency; the noise output voltage of the discriminator is then given by

$$e_{no} = k f_r \frac{N}{C} \quad , \qquad \text{and since the IF bandwidth}$$

restricts f_n so that $(f_c - \frac{B}{2}) \leq f_n \leq (f_c + \frac{B}{2})$, the noise output voltage lies within the range $\frac{B}{2} \geq f_r \geq 0$. This noise spectrum is plotted* in Fig. 3. However, this entire noise spectrum does not appear in the final receiver output channel. In a single channel system, a low-pass audio system restricts the output to the frequency range zero to F_a; in a multichannel system with frequency-spaced sub-carriers, the channel filters restrict the range to that between two frequencies, say f_1 and f_2. If figure 3 is re-plotted on the same abcissa but with squared ordinate values, the total power (noise) in the output, over the range f_1 to f_2, will be proportional** to the area under

*Rice (16)
**Ballantine (3)

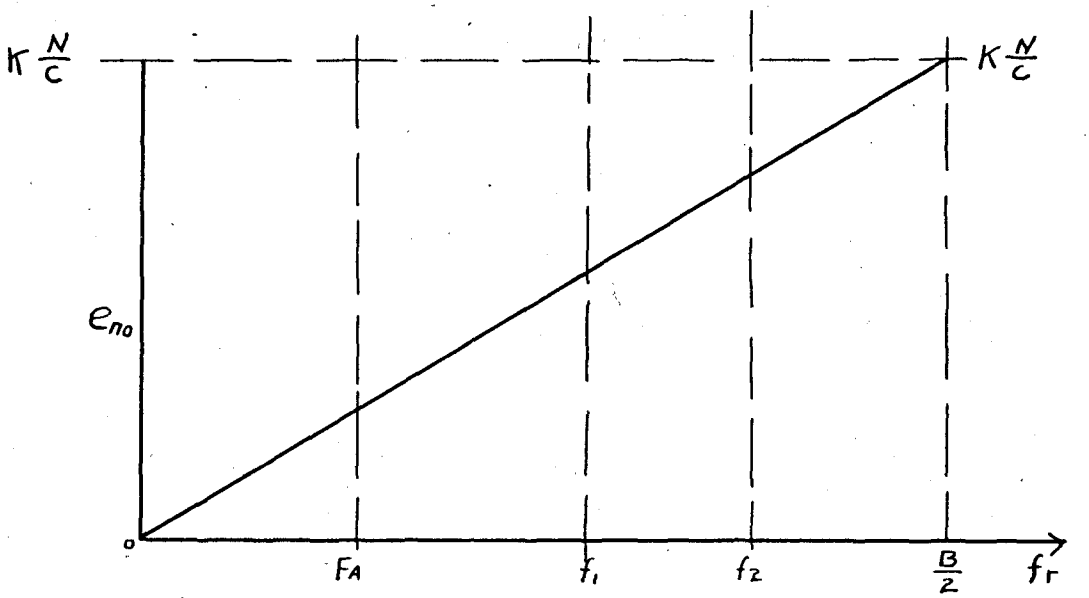


Figure 3.

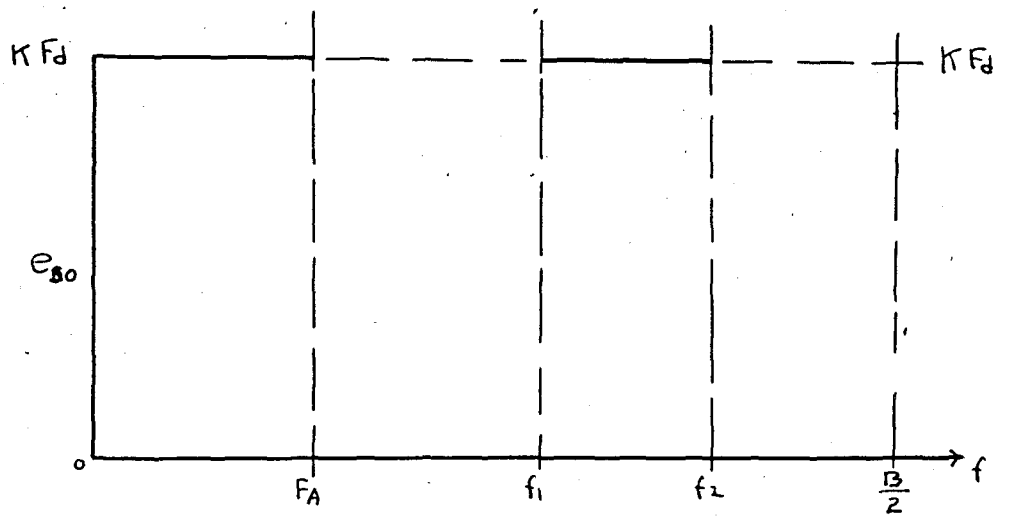


Figure 4.

this curve between the limits f_1 and f_2. Hence, in a multi-channel system of the frequency division type, using frequency modulation of the carrier, the S/N ratio of any channel will be a function of the subcarrier frequency. In most cases, it is more desirable to have essentially the same S/N ratio for each channel. This can be accomplished by properly proportioning the amount that each channel is allowed to modulate the carrier. As will be shown, this difficulty does not exist when the carrier is phase - instead of frequency-modulated.

The peak frequency deviation due to the desired intelligence is F_d; the signal output voltage of the discriminator is then given by

$e_{so} = KF_d$, plotted in Fig. 4. Again, the total signal power in the output is proportional to the area under the squared-ordinate curve over the output frequency range.

Calculating the S/N ratio for a single channel system, 100% modulation, with an output frequency range from zero to F_a ; equivalent load resistance R :

Total noise power within the range $0 \leq f_r \leq F_a$ is

$$P_n = \frac{1}{R}\int_0^{F_a} e_n^2 \, df_r = \frac{1}{R}\int_0^{F_a} k^2\left(\frac{N}{C}\right)^2 f_r^2 \, df_r = \frac{k^2}{R}\left(\frac{N}{C}\right)^2 \frac{F_a^3}{3}$$

Total signal power within the range $0 \leq f_p \leq F_a$ is

$$P_s = \frac{1}{R}\int_0^{F_a} e_s^2\, df_p = \frac{1}{R}\int_0^{F_a} k^2 F_d^2\, df_p$$

$$= \frac{k^2}{R} F_d^2 F_a$$

hence the ratio of signal power to noise power is equal to $3\,(C/N)^2\,(F_d/F_a)^2$, from which finally we have

$$\frac{S}{R} = \sqrt{3}\,\frac{C}{N}\,\frac{F_d}{F_a} \qquad \text{or,} \quad G = \sqrt{3}\,\frac{F_d}{F_a}$$

For a multi-channel system, the S/N ratio of one channel extending over the frequency range f_1 to f_2 in the output is derived as follows:

Total noise power within the range $f_1 \leq f_r \leq f_2$ is

$$P_n = \frac{1}{R}\int_{f_1}^{f_2} e_n^2\, df_r = \frac{k^2}{R}\left(\frac{N}{C}\right)^2 \frac{f_2^2 - f_1^2}{3}$$

and

Total signal power within the range $f_1 \leq f_p \leq f_2$ is

$$P_s = \frac{1}{R}\int_{f_1}^{f_2} e_s^2\, df_p = \frac{k^2}{R} F_d^2 (f_2 - f_1)$$

giving a signal power to noise power ratio of

$$3\left(\frac{C}{N}\right)^2 \frac{F_d^2}{f_2^2 + f_1 f_2 + f_1^2}$$

and since any one channel cannot be allowed to modulate the carrier 100%, we have finally for one channel

$$G = \sqrt{3}\, M_c \frac{F_d}{\sqrt{f_2^2 + f_1 f_2 + f_1^2}}$$

where M_c is the amount one channel can modulate the carrier, compared to 100%. Note that this G is that which exists at the input to the individual channel detector in a frequency division system. For the single channel system the expressions for the other performance factors become:

$$B = B_t = 2F_a \left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$P = P_t = 16\ KT\ F_a \left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$M = \frac{1}{32\ KT\ F_a^2 \left(\frac{Q}{3}\right)^{2/3}}$$

$$PI = \frac{20}{3} \log(1.732\ Q)$$

3. Phase Modulation (FM)

A phase modulated carrier wave is expressed by

$e_c = C \sin(w_c t + \beta \cos w_p t)$, where $\beta = \theta_d$, the maximum phase deviation.

Except for the value of β this is indistinguishable from a frequency modulated wave, and can be analyzed as such. On this basis, the same triangular noise spectrum (Fig. 3) is obtained. However, in this case the signal spectrum differs, and is as follows:

The equivalent FM produced by modulation is derived from the expression for instantaneous frequency

$$f = \frac{1}{2\pi} \frac{d\theta}{dt} = f_c - \theta_d f_p \sin w_p t$$

The frequency deviation produced by signal modulation is not equal to F_d but is now equal to $\theta_d f_p$, a function of the modulating frequency itself. The maximum frequency deviation will be equal to $\theta_d F_a$ so we can write for the maximum instantaneous deviation due to signal modulation $(\frac{F_d}{F_a}) f_p$ and the corresponding discriminator signal output voltage then is

$$e_s = K \frac{F_d}{F_a} f_p$$

The discriminator noise output voltage is again

$$e_n = K \frac{N}{C} f_r$$

Both spectra are seen here to be triangular in form. For a single channel system we have

Total noise power within the range $0 \leq f_r \leq F_a$ is

$$P_n = \frac{1}{R} \int_0^{F_a} e_s^2 \, df_r = \frac{k^2}{R} \left(\frac{N}{C}\right)^2 \frac{F_a^3}{3}$$ and

Total signal power within the range $0 \leq f_p \leq F_a$ is

$$P_s = \frac{1}{R} \int_0^{F_a} e_s^2 df_p = \frac{k^2}{R} \left(\frac{Fd}{Fa}\right)^2 \frac{F_a^3}{3}$$ from which

$$\frac{S}{N} = \frac{C}{N} \frac{Fd}{Fa}$$ and $$G = \frac{Fd}{Fa}$$

For a multichannel system, designating the highest frequency component of the complex modulating signal as applied to the carrier by F_h, the S/N ratio of one channel extending over the range f_1 to f_2 in the detector output is found as follows:

Total noise power over the range $f_1 \leq f_r \leq f_2$ is

$$P_n = \frac{1}{R} \int_{f_1}^{f_2} e_n^2 \, df_r = \frac{k^2}{R} \left(\frac{N}{C}\right)^2 \frac{f_2^2 - f_1^2}{3}$$ and

Total signal power over the range $f_1 \leq f_p \leq f_2$ is

$$P_s = \frac{1}{R}\int_{f_1}^{f_2} e_s^2 \, df_p = \frac{k^2}{R}\left(\frac{F_d}{F_h}\right)^2 \frac{f_2^2 - f_1^2}{3}$$ from which

$$\frac{S}{N} = \frac{C}{N}\frac{F_d}{F_h} M_c \qquad \text{and} \qquad G = \frac{F_d}{F_h} M_c$$

Again, this value of G is that obtaining at the input to the individual channel detector.

For the single channel system the expressions for the other performance factors become.

$$B = B_t = 2F_a \, Q^{\frac{1}{3}}$$

$$P = P_t = 16 \, KT \, F_a \, Q^{\frac{1}{3}}$$

$$M = \frac{1}{32 \, KT \, F_a^2 \, Q^{2/3}}$$

$$PI = \frac{20}{3} \log Q$$

CHAPTER IV

TIME DIVISION SYSTEMS

1. PAM-AM

In this system, each channel is furnished with a pulse sub-carrier of frequency $f_p = k_1 F_a$, where F_a is the highest intelligence frequency within the channel. The value of k_1 may be as low as 2 if an infinitely sharp low-pass filter is used; a value of $k_1 = 3$ is commonly used in practise.

The modulating signal is composed of the modulated pulse sub-carriers of n channels, and hence has a fundamental pulse frequency of $p_r = nk_1 F_a$. The pulses of each channel have a duty factor which is the product of pulse duration and channel pulse frequency: $D = f_p \tau$. The modulating wave has a duty factor $D_o = n\tau f_p = \tau p_r$.

Assuming that the IF bandwidth is no greater than that actually required to pass the modulated wave, the modulating signal is recovered at the group detector output with a S/N ratio equal to the C/N ratio at the detector input. For the same power, the C/N ratio is increased by the factor $\frac{1}{\sqrt{D_o}}$. The detector output is sampled at the rate f_p. The signal output in any channel is a DC component proportional to the sum of the areas under the series of pulses, hence proportional to the channel duty factor. On the other hand, the noise components in the

consecutive pulses are random in phase; the total noise power in one channel is proportional to the square-root of the sum of the squared areas under the pulses, and therefore the noise voltage is proportional to the square-root of the channel duty factor D.

$$G = \frac{1}{\sqrt{D_o}} \frac{D}{\sqrt{D}} = \frac{1}{\sqrt{n}}$$

Using minimum length pulses spaced as indicated in Fig. 5a, the minimum required bandwidth for the system is $12nF_a$. The choice of the pulse spacing is dictated by cross-modulation considerations, discussed in the section devoted to Distortion.

The expressions for the other performance factors become:

$$B = B_{min} = 12n\ F_a$$

$$P_c = 16\ KT\ F_a\ n\ Q$$

$$M = \frac{1}{384\ KT\ F_a^2\ n^2\ Q}$$

$$PI = -10\ \log(n)$$

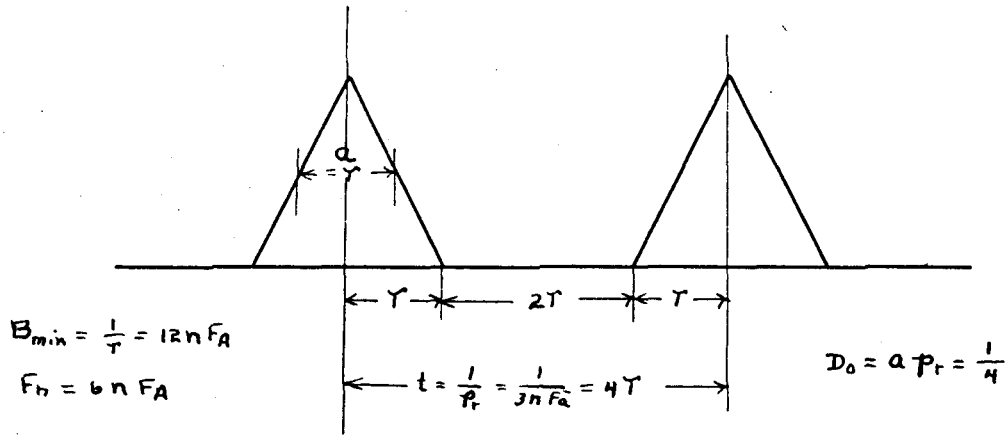
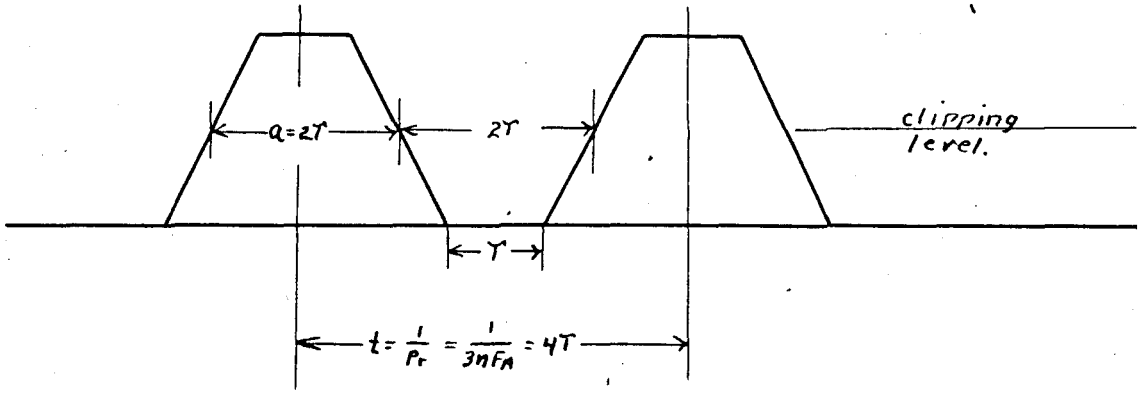


Figure 5(a)



$F_H = 12nF_A$ $D_o = a P_r = \frac{1}{2}$ $B_{min} = 24nF_A$

Figure 5(b)

2. PAM-FM

The modulating signal is similar to that for PAM-AM, and is recovered at the detector output with a signal-to-noise ratio proportional to $(3)^{\frac{1}{2}} F_d/F_h$. Sampling of this signal to separate the channels produces again the factor $(D)^{\frac{1}{2}}$, giving

$$G = (3)^{\frac{1}{2}} (D)^{\frac{1}{2}} F_d/F_h$$

This expression indicates the desirability of using greater pulse widths than minimum, to increase the duty factor. With clipping at the 50% level in the receiver, utilizing only the flat top portion of the received pulses, the modulating signal duty factor can be increased to $D_o = 0,5$ (see Fig. 5b.) The minimum required bandwidth is in this case $24nF_a$, and F_h is $12nF_a$. The channel duty factor $D = 1/2n$. Substituting and clearing,

$$G = \frac{1}{e\sqrt{6}} \frac{B}{F_a n^{3/2}}$$ and

$$B = B_t = 4nF_a (12 Q)^{\frac{1}{3}}$$

$$P = P_t = 32KTnF_a (12 Q)^{\frac{1}{3}}$$

$$M = \frac{1}{128 KT n^2 F_a^2 (12 Q)^{2/3}}$$

$$PI = \frac{20}{3} \log\left(\frac{Q}{4\sqrt{6}\, n^{3/2}} \right)$$

3. PPM-AM

In this system, the intelligence is impressed upon the channel pulse train in such a manner as to vary the position of the individual pulses from their median or no-signal position. This procedure produces in effect an equivalent frequency or phase-modulation of the pulse train sub-carrier. For a large number of channels, the equivalent frequency deviation becomes so small that the practical production and analysis of modulation is more conveniently conducted on the basis of equivalent phase modulation.

Consider a channel pulse train of frequency f_p and period $1/f_p$; the angular separation between pulses is 2π. For an n-channel system, the angular spacing between pulses is $2\pi/n$. The maximum angular deviation allowed any pulse is half this spacing, to prevent overlapping; to reduce inter-channel cross-modulation with moderate filter requirements, it is common practice to limit the maximum deviation to two-thirds the theoretical limit. On this ground,

$$\theta_m = \frac{2\pi}{3n}$$

At the detector output, the modulating signal is recovered with a S/N ratio proportional to $1/\sqrt{n}$, as for PAM-AM. In this instance, however, the noise voltage has a different effect. It does not appear directly in the

final output, but affects the channel signal by introducing an equivalent phase shift of the individual pulse. The effect is indicated in Fig. 6, for minimum length pulses and clipping at 50% level. The maximum time displacement t_n produced by the noise occurs when the peak of the noise pulse corresponds to the beginning of the signal pulse and amounts to $\tau/2$. The corresponding phase displacement is equal $\theta_n = 2\pi f_p t_n = \frac{f_p \tau}{B}$. Then

$$G = \frac{1}{\sqrt{n}} \frac{\frac{2\pi}{3n}}{\frac{\pi f_p}{B}} = \frac{2}{3} \frac{1}{n^{3/2}} \frac{B}{f_p} = \frac{2}{9} \frac{B}{F_a n^{3/2}}$$

The required minimum bandwidth is $36nF_a$.

The Improvement G could be increased by at least $(2)^{\frac{1}{2}}$ if there were a practicable means of using the center of gravity of the pulse as the reference point. The complex circuitry required to do this has resulted in the more simple method of using only the leading edge of the pulse, with combined limiting and clipping.

The expressions for the other performance factors become:

$$B = B_t = 3nF_a \ (1.5\ Q)^{\frac{1}{2}}$$

$$P = P_t = 36KTnF_a\ Q$$

$$M = \frac{1}{108\ KT\ n^2\ F_a^2\ (1.5\ Q)^{\frac{1}{2}}}$$

$$PI = -10 \log \left(\frac{9n}{4}\right)$$

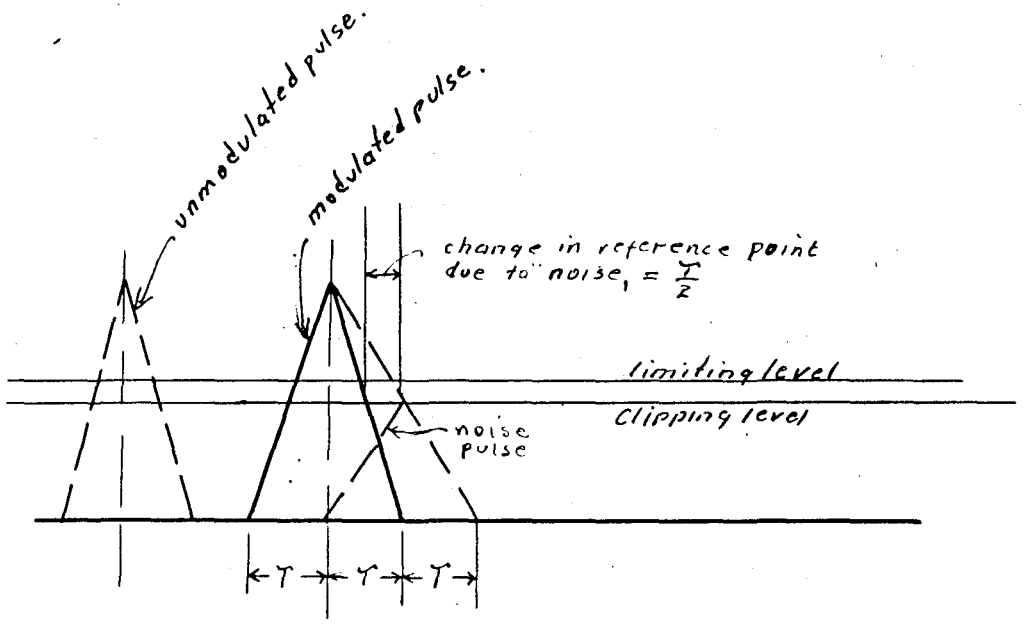

unmodulated pulse.
modulated pulse.
change in reference point due to noise, = T/2
limiting level
clipping level
noise pulse
T
T
T

Figure 6.

4. PPM-FM

The modulating signal is recovered at the group detector output with a S/N ratio proportional to $(3)^{\frac{1}{2}}\ (D)^{\frac{1}{2}}\ F_d/F_h$, as in the PAM-FM system. The noise voltage does not appear directly in the output, but acts to produce an equivalent phase modulation on the signal pulses. The effect is again illustrated by Fig.6. The minimum length of pulse in the detector output is $1/2F_h$, which corresponds to twice the time displacement possible due to a noise pulse; the equivalent phase displacement is

$2\pi f_p/4F_h = \pi/12n = \theta_n \qquad D = \frac{3F_a}{B}$

The displacement due to modulation is again $2\pi/3n$. Substituting, we have

$$G = (3)^{\frac{1}{2}}\ (D)^{\frac{1}{2}} \left(F_d/F_h\right)\frac{\theta_m}{\theta_n} = \frac{2}{3n}\ \left(\frac{B}{F_a}\right)^{\frac{1}{2}}$$

The minimum required bandwidth is $36nF_a$.

There is no advantage to increasing pulse duration since the valve of θ_m must be decreased in the same proportion.

The expressions for the other performance factors become:

$$B = B_t = 3nF_a\ (0.5\ Q)^{\frac{1}{2}}$$

$$P = P_t = 24\ KTnF_a\ (0.5\ Q)^{\frac{1}{2}}$$

$$M = \frac{1}{36\ KT\ n^2\ F_a^2\ Q}$$

$$PI = 5\log\ (\frac{8\ Q}{9\ n^2})$$

5. PWM-AM

The modulating signal for each channel is impressed upon the channel pulse subcarrier in such a manner as to vary the width (duration) of the individual pulse in accordance with the intelligence to be transmitted. This change in pulse width is commonly measured from one side of the pulse which is held fixed, rather than both ways from the center of the unmodulated pulse. In this fashion noise can affect only one side of the pulse instead of both sides, since the receiver can be gated at the fixed side by a locally generated pulse.

At the group detector output the complex modulating signal is recovered with a signal-to-noise ratio proportional to $\frac{1}{\sqrt{D_o}}$ when the average radiated power of the PWM-AM system equals the carrier power of the single channel AM system and D_o is the average carrier duty factor. Variation of the pulse width is essentially a variation of the pulse duty factor. On demodulation of the channels, the signal voltage output is proportional to the change of pulse duty factor caused by the modulation. The noise voltage output of a channel is proportional to the square-root of the change of the pulse duty factor caused by the presence of the noise. The square-root is involved because over any interval of time the noise effect on one pulse has no correlation with the noise effect on any other pulse and the total effect is

therefore an rms summation. Referring to Fig. 7, the change in pulse duty factor due to the noise is (τf_p). From Fig. 8, the change in pulse duty factor due to the modulation is $(\frac{1}{nf_p} - 3\tau)f_p$. The average carrier duty factor is given by $D_o = \frac{B - 6nF_a}{2B}$. Substitution of these into

$$G = \frac{1}{\sqrt{D_o}} \frac{(\Delta D) \text{ due to modulation}}{\sqrt{(\Delta D) \text{ due to noise}}}$$

$$G = \sqrt{\frac{2}{3}} \left[\frac{B - 9nF_a}{F_a^{\frac{1}{2}} n \sqrt{B - 6nF_a}} \right]$$

For wide band operation this expression approaches the simpler form

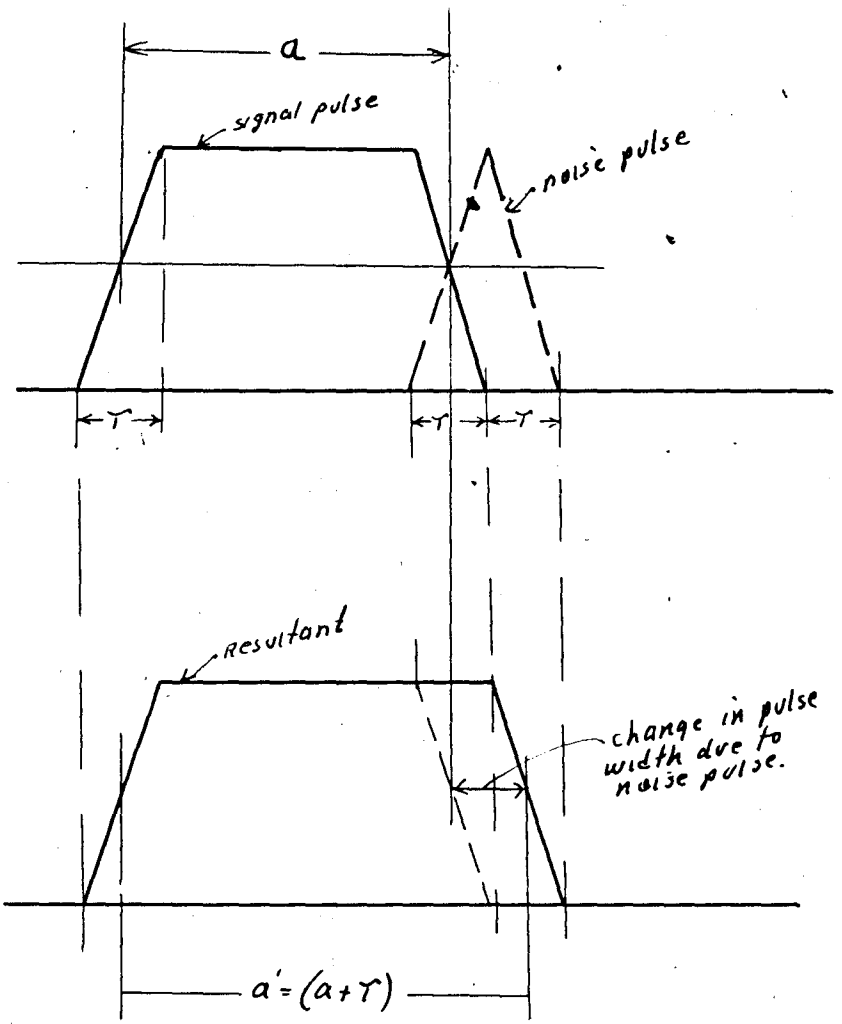
$$G = \sqrt{\frac{2}{3}} \frac{B^{\frac{1}{2}}}{n F_a^{\frac{1}{2}}} \qquad \text{and}$$

$$B = B_t = 3nF_a Q^{\frac{1}{2}}$$

$$P = P_t = 12 KTnF_a Q^{\frac{1}{2}}$$

$$M = \frac{1}{36 KT n^2 F_a^2 Q}$$

$$PI = 5 \log\left(\frac{16 Q}{9 n^2}\right)$$



$$\Delta D = (\Delta a) f_p = \tau f_p$$

Figure 7.

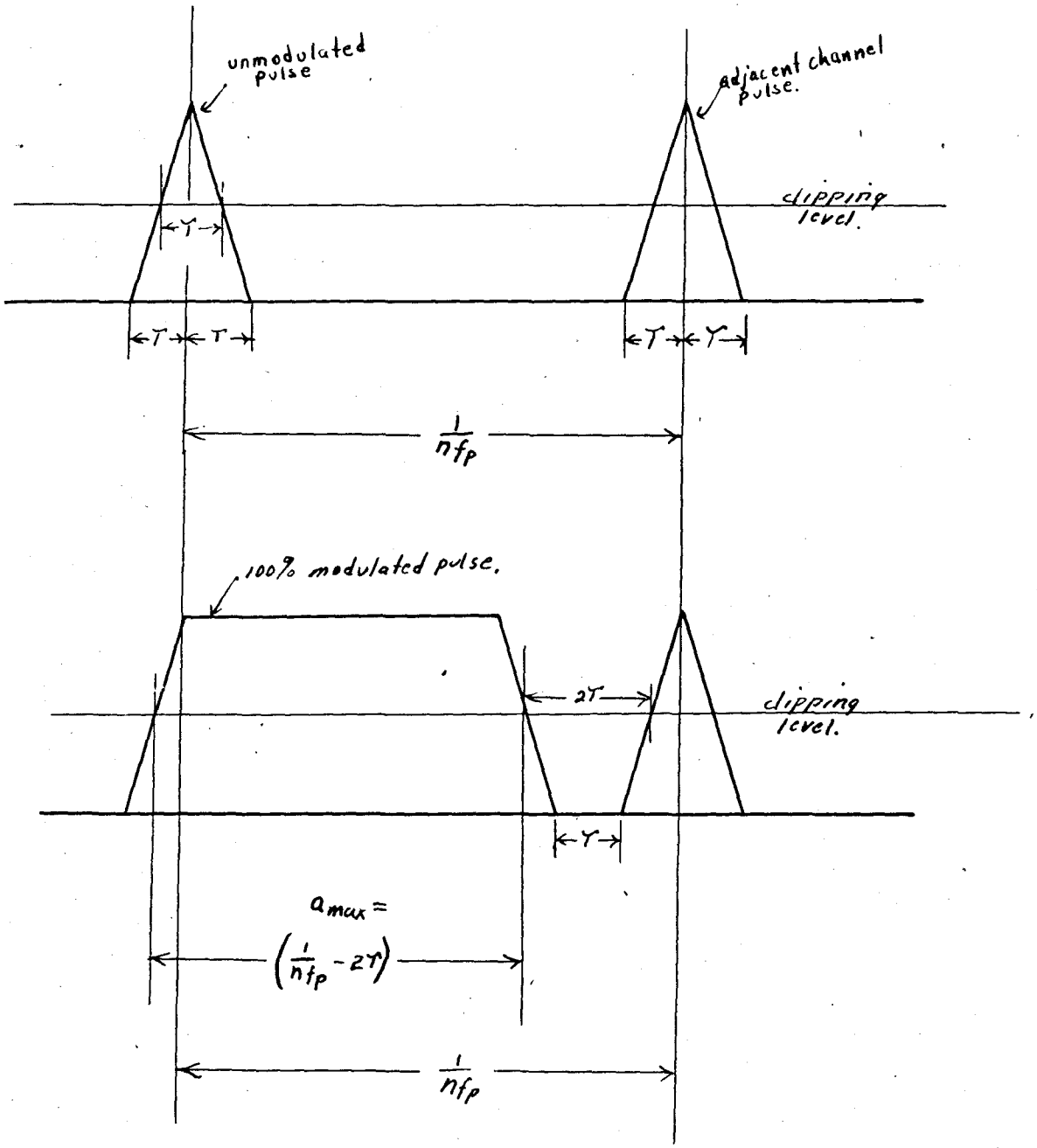

unmodulated pulse
adjacent channel pulse.
clipping level.
τ
τ
τ
τ
τ
1/nfp
100% modulated pulse.
2τ
clipping level.
τ
amax =
(1/nfp - 2τ)
1/nfp

Figure. 8.

6. PWM-FM

The modulating signal is recovered at the group detector output with a S/N proportional to $(3)^{\frac{1}{2}} F_d/F_h$. The effect of the noise is as in PWM-AM, and we have

$$G = \sqrt{3}\,\frac{F_d}{F_h}\,\frac{(\Delta a)_{mod}\ f_p}{\left[(\Delta a)_{noise}\ f_p\right]^{\frac{1}{2}}}$$

The minimum required bandwidth is $24nF_a$, and the minimum value of F_h is $12nF_a$. The minimum length of pulse in the detected output is $1/2F_h$, equals the change in pulse length possible due to noise, equals $1/8nf_p$, equals τ. The change in pulse duration due to signal is $(1/nf_p - 3\tau)$ equals $5/8nf_p$. Substituting and simplifying,

$$G = \frac{5}{16\sqrt{6}}\,\frac{B}{F_a\ n^{3/2}} \qquad \text{and}$$

$$B = B_t = 16nF_a\ (\frac{3\ Q}{50})^{\frac{1}{3}}$$

$$P = P_t = 128\ KTnF_a\ (\frac{3\ Q}{50})^{\frac{1}{3}}$$

$$M = \frac{1}{1048\ KT\ n^2\ F_a^2\ (\frac{3\ Q}{50})^{2/3}}$$

$$PI = \frac{20}{3}\log(\frac{2.92\ Q}{n^{3/2}})$$

7. PNM-XX and PCM-XX

These systems offer a theoretically infinite S/N ratio when operated above the threshold level. They are essentially the same; PNM (Pulse Number Modulation) is a simplified version of PCM (Pulse Code Modulation). The PCM system operates as follows:

The modulating signal is scanned (sampled) at a rate (kf_p), allowing (k) pulses per sample instead of one, as in the pulse systems previously described. The intelligence, instead of modulating pulse shape or phase, acts to code the group of k pulses in a predetermined manner. The coding method based on the binary system of numbers is most readily adaptable. Using this method, the absence of a pulse indicates a cipher, and the presence of a pulse denotes unity or the number 1. The possible code groups for $K=3$ are indicated in Fig. 9; there are eight possible combinations which can be made to correspond to eight levels of the mondulating information. For $K=7$, there are 128 possible gradations of signal level. In general, there are 2^k groups for any value of k. PNM differs from PCM in that the former involves a coding process which considers only the number of pulses in a group without regard to their sequence in time. The simplification in circuitry is hence accompanied by a great decrease in the possible number of signal level gradations which can be coded b y a group of

K pulses. PCM is taken up in more detail in a later section, but we can here derive the expressions for the performance factors other than the Improvement G. Since $G = \infty$ for operation above threshold and is independent of bandwidth it would be wasteful to use greater than the minimum required bandwidth for operation. For a seven-digit PCM system this results in for PCM-AM :

$$B = B_{min} = 63nF_a$$

$$P = P_t = 168KTnF_a$$

$$M = \frac{1}{8064\ KT\ n^2\ F_a^2}$$

$$PI = 10 \log\left(\frac{2\ Q}{21\ n}\right)$$

and for PCM-FM :

$$B = B_{min} = 63nF_a$$

$$P = P_t = 504\ KTnF_a$$

$$M = \frac{1}{31752\ KT\ n^2\ F_a^2}$$

$$PI = 10 \log\left(\frac{2\ Q}{63\ n}\right)$$

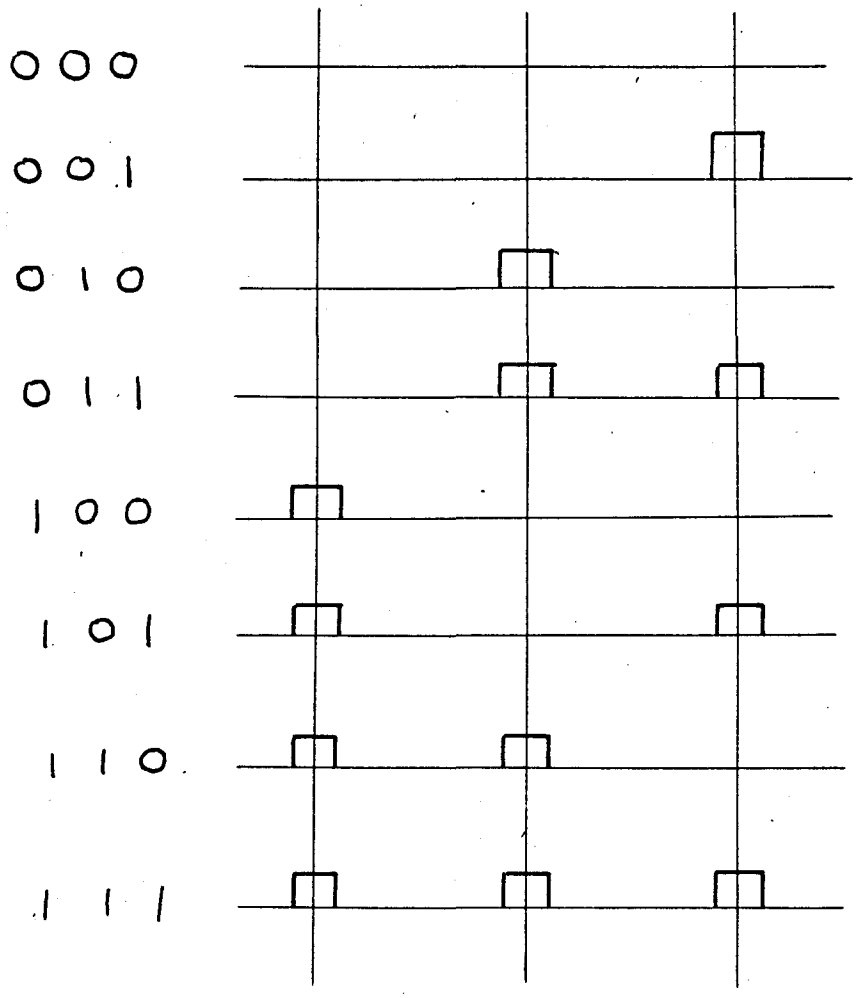


Figure 9.

CHAPTER V

FREQUENCY DIVISION SYSTEMS

1. AM-AM

A group of continuous wave subcarriers appropriately spaced in frequency are amplitude modulated by their respective intelligence inputs. The combined group of modulated subcarriers is then applied as a complex modulating signal to amplitude modulate the carrier.

We can write in general for AM systems

$$\frac{S}{N} = \frac{C}{N})_{2F_a}$$

where the subscripts indicate the frequency bands over which the noise voltage is measured. For a double AM system this becomes

$$\frac{S}{N})_{F_a} = \frac{C_s}{N})_{2F_h} = \frac{C_s}{N})_{F_h}(\frac{F_h}{2F_a})^{\frac{1}{2}}$$

where $\frac{C_s}{N})_{F_h}$ represents the subcarrier-to-noise ratio over the frequency band zero to F_h ; ie, the output of the first detector. But

$$\frac{C_s}{N})_{F_h} = \frac{1}{2n}\frac{C}{N})_B = \frac{1}{2n}\frac{C}{N})_{2F_a}(\frac{2F_a}{B})^{\frac{1}{2}}$$

and from this and substituting $B = 2F_h$ we obtain

$$\frac{S}{N} = \left(\frac{F_h}{2F_a}\right)^{\frac{1}{2}} \frac{1}{2n} \frac{C}{N})_{2F_a} \left(\frac{2F_a}{B}\right)^{\frac{1}{2}} = \frac{1}{2n\sqrt{2}} \frac{C}{N})_{2F_a} \quad \text{or,}$$

$$G = \frac{1}{2n\sqrt{2}}$$

This expression assumes linear addition of the modulated subcarriers in forming the complex modulating signal; each subcarrier can be allowed to modulate the carrier by an amount $\frac{1}{2n}$ compared to 100%. For the case where $n=1$, we have

$$G = \frac{1}{2n\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

The factor $\frac{1}{\sqrt{2}}$ represents the loss in signal to noise ratio due to doubling the RF noise acceptance band; it is now $4F_a$ instead of the value $2F_a$ for the single AM system. This loss factor will enter in all Frequency Division Systems not involving single side-band transmissions. The factor $(\frac{1}{2})$ represents the loss due to the transmission of the subcarrier itself.

The above derivation is probably pessimistic in its assumption of linear addition of the subchannels in forming the complex modulating signal, particularly for systems in which the number of channels is quite large. Under these conditions, and assuming that the intelligence in any one channel is not related to the intelligence in any other channel, the complex modulating signal formed by the addition of all the subchannels can be regarded as similar to a noise

voltage. From this viewpoint, the modulating signal will have a crest factor of four and will consist of the n components added in rms fashion. The rms value of n components of equal peak amplitudes E_n is proportional to $\frac{1}{\sqrt{2}}\sqrt{n}$. The peak value of the complex modulating signal is then given as

$$E_m = 4\frac{1}{\sqrt{2}}\sqrt{n}\,E_n$$

The amount of modulation any one channel can produce upon the carrier is therefore

$$\frac{E_n}{E_m} = \frac{\sqrt{2}}{4\sqrt{n}}$$

instead of $\frac{1}{2n}$ as before, and hence

$$G = \frac{1}{\sqrt{2}}\,\frac{\sqrt{2}}{4\sqrt{n}} = \frac{1}{4\sqrt{n}}$$ and also for large n,

$$B = B_{min} = 2nf_w$$

$$P = P_c = 128\ KT\ n^2\ F_a\ Q$$

$$M = \frac{1}{256\ KT\ n^3\ F_a\ f_w\ Q}$$

$$PI = -20\log(4\sqrt{n})$$

2. AM-PM

The modulating signal is the same as that described for the case of AM-AM, but is now applied to the carrier to produce Phase Modulation (PM). The modulating signal is recovered at the group detector output with a signal-to-noise ratio proportional to $\frac{F_d}{F_h} M_c$ as previously shown.

For small values of n, $M_c = \frac{1}{2n}$

For large values of n, $M_c = \frac{\sqrt{2}}{4\sqrt{n}}$

Designating the frequency spacing between subcarriers by f_w which will be $2F_a$ plus a small guard space, the modulating signal occupies a band given by

$$F_h = (n - 1)f_w + 2F_a \cong nf_w$$

The doubled RF noise acceptance bandwidth again produces a factor of $\frac{1}{\sqrt{2}}$. Substituting the above values, and remembering that $F_d = \frac{B}{2}$, we obtain

$$G = \frac{B}{4\sqrt{2}\, n^2 f_w} \qquad \text{for small n}$$

$$G = \frac{B}{8 f_w n^{3/2}} \qquad \text{for large n}$$

$$B = B_t = 4n (2 F_a f_w^2 Q)^{\frac{1}{3}}$$

$$P = P_t = 32 KT n (2 F_a f_w^2 Q)^{\frac{1}{3}}$$

$$M = \frac{1}{128 KT n^2 (2 F_a f_w^2 Q)^{2/3}}$$

$$PI = \frac{20}{3} \log\left(\frac{F_a Q}{4 f_w n^{3/2}} \right)$$

3. SS-AM

Subcarriers appropriately spaced in frequency are amplitude modulated by their respective intelligence inputs to produce single-side-band, suppressed carrier outputs for each channel. These outputs are then combined to form the modulating signal which amplitude modulates the carrier.

Comparing this system to the simple AM-AM system we note the following:

Eliminating one channel side-band allows doubling the other, maintaining the same signal output, but requiring only one half the RF bandwidth. The signal-to-noise ratio is therefore improved by the factor $(2)^{\frac{1}{2}}$. Eliminating the subcarrier allows each subchannel to modulate the carrier twice as much. The total improvement over the AM-AM case is $2\sqrt{2}$.

$$G = 2\sqrt{2}\,\frac{1}{2n\sqrt{2}} = \frac{1}{n} \qquad \text{for small n}$$

$$G = 2\sqrt{2}\,\frac{1}{4\sqrt{n}} = \frac{1}{\sqrt{2n}} \qquad \text{for large n}$$

The expressions for the other performance factors become:

$$B = B_{min} = nf_w$$

$$P = P_c = 32\ KTnF_a\ Q$$

$$M = \frac{1}{32\ KT\ n^2\ F_a\ f_w\ Q}$$

$$PI = -10\log(2n)$$

4. SS-PM

This system is similar to SS-AM except that the complex modulating signal is applied to phase-modulate the carrier. Compared to AM-PM we note the following:

Eliminating one side-band of each channel allows doubling the other, maintaining the same signal output but requiring only half the bandwidth. The signal-to-noise ratio is thereby improved by the factor $\sqrt{2}$. Eliminating the subcarrier allows each subcarrier to modulate the carrier twice as much, giving a comparative improvement by the factor 2. Furthermore, the halving of the modulating signal bandwidth F_h effectively doubles the PM modulation index $\frac{Fd}{Fh}$, producing a further improvement of 2. The total improvement compared to the AM-PM system is therefore $4\sqrt{2}$, and we have

$$G = 4\sqrt{2}\,\frac{B}{4\sqrt{2}\,n^2 f_w} = \frac{B}{n^2 f_w} \qquad \text{for small n}$$

$$G = 4\sqrt{2}\,\frac{B}{8 f_w n^{3/2}} = \frac{B}{\sqrt{2}\,n^{3/2} f_w} \qquad \text{and also for large n}$$

$$B = B_t = n(4F_a f_w^2 Q)^{1/3}$$

$$P = P_t = 8\,KTn(4F_a f_w^2 Q)^{1/3}$$

$$M = \frac{1}{8\,KT\,n^2\,(4F_a f_w^2 Q)^{2/3}}$$

$$PI = \frac{20}{3}\log\left(\frac{\sqrt{2}\,F_a Q}{f_w n^{3/2}}\right)$$

5. FM-AM

Frequency spaced subcarriers are frequency-modulated by their respective intelligence inputs; the modulated subcarriers are then combined to form the complex modulating signal applied to amplitude-modulate the carrier.

Recovery of the modulated subcarrier is accomplished with a factor proportional to $\frac{Mc}{\sqrt{2}}$, where M_c is the relative amount that a subchannel can modulated the carrier and factor $\frac{1}{\sqrt{2}}$ is the loss due to the double RF noise acceptance band compared to the single channel AM system.

Demodulation of the individual channel is accomplished with a factor $\sqrt{3}\,\frac{f_{dn}}{F_a}$ where f_{dn} denotes the peak deviation of the subcarrier caused by the intelligence, $= \frac{B}{4n}$. Hence,

$$G = \sqrt{\frac{3}{2}}\, M_c\, \frac{f_{dn}}{F_a}$$

As previously derived, the value of M_c is $\frac{1}{n}$ for linear addition of the subcarriers and $\frac{\sqrt{2}}{4\sqrt{n}}$ for rms addition. Substituting,

$$G = \sqrt{\frac{6}{8}}\,\frac{B}{F_a\, n^2} \qquad \text{for small n}$$

$$G = \frac{\sqrt{3}}{16}\,\frac{B}{F_a\, n^{3/2}} \qquad \text{for large n}$$

The expressions for the remaining performance factors become for large n:

$$B = B_t = 8nF_a \left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$P = P_t = 64KTnF_a \left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$M = \frac{1}{512\ KT\ n^2\ F_a^2 \left(\frac{Q}{3}\right)^{2/3}}$$

$$PI = 20\ \mathrm{Log}\left(\frac{3\ Q}{8\ n^{3/2}}\right)$$

6. FM-PM

The modulating signal is similar to that for the FM-AM system previously described but in this system it is applied to phase-modulate the carrier. The improvement writes as

$$G = \frac{1}{\sqrt{2}}\sqrt{3}\,\frac{f_{dn}}{F_A}\,\frac{F_d}{F_h}\,M_c \qquad \text{where}$$

$M_c = \frac{1}{n}$ for linear addition of channels

$M_c = \frac{\sqrt{2}}{4\sqrt{n}}$ for rms addition of channels

$F_h = 2nf_{dn}$

$F_d = \frac{1}{2}B$ and substituting,

$$G = \sqrt{\frac{3}{2}}\,\frac{B}{4\,n^2\,F_a} \qquad \text{for small } n$$

$$G = \frac{\sqrt{3}}{16}\,\frac{B}{F_a\,n^{3/2}} \qquad \text{for large } n, \text{ and}$$

$$B = B_t = 8nF_a\left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$P = P_t = 64KTnF_a\left(\frac{Q}{3}\right)^{\frac{1}{3}}$$

$$M = \frac{1}{512\,KT\,n^2\,F_a^2\left(\frac{Q}{3}\right)^{2/3}}$$

$$PI = \frac{20}{3}\log\left(\frac{\sqrt{3}\,Q}{8\,n^{3/2}}\right)$$

SYSTEM	$M \times 10^{-4}$	P (44 watts)	B (mc)	G	P. I. (db)
PCM-AM	26.1	0.2015	18.9	∞	+ 19.82
PNM-AM	26.1	0.2015	18.9	∞	+ 19.82
PCM-FM	8.72	0.605	18.9	∞	+ 15.02
PNM-FM	8.72	0.605	18.9	∞	+ 15.02
SS-PM	1.71	5.85	10	1.765	+ 5.17
PPM-AM	0.303	33.0	10	0.742	-2.35
PWM-FM	0.100	100.5	10	0.452	-7.19
PWM-AM	0.0813	123	10	0.385	-8.06
PPM-FM	0.0813	123	10	0.385	-8.06
FM-PM	0.0713	140	10	0.360	-8.63
FM-AM	0.0713	140	10	0.360	-8.63
SS-AM	0.0326	3840	0.8	0.0707	-23.0
PAM-FM	0.0155	643	10	0.170	-15.25
PAM-AM	0.0145	145	3.6	0.100.	-20.0
AM-AM	0.0023	30700	1.4	0.025	-32.04
AM-PM	0.000017	6.03×10^6	10	0.0018	-54.97

Table 1.

CHAPTER VI

DISTORTION AND BANDWIDTH

1. Introduction.

Ordinary non-linear distortion effects upon the individual channels can be held within acceptable limits by the normal techniques employed for its minimization in single channel equipment. The most distrubing distortion occurring in multi-channel systems is interchannel distortion, the appearance in a channel output of undesired energy from one or more other channels. If conditions are such that small amounts of energy from many channels appear in the output of a channel, the effect is to produce a background level similar to noise which may be considered as a degradation of the signal-to-noise ratio of that channel. This condition is not as serious as that in which larger amounts of energy from one or two adjacent channels appear in the desired output, for the interference has the character of conflicting intelligence rather than the more neutral background of random noise. The production of interchannel interference is affected by different factors in the frequency division systems and the time division systems.

It should be realized that a system which provides satisfactory point-to-point communication from a distortion point of view may be wholly useless for relay work. The cumulative distortion over a chain involving two or more

individually satisfactory equipments may well render the complete system worthless.

2. Frequency Division Systems.

Curvature of the input-output characteristics of the amplifiers will produce non-linear distortion of the intelligence. While the amount of intelligence distortion thus produced may be within tolerable limits, the harmonics produced may lie within other channels and may be of sufficient number and amplitude to be unacceptable. Furthermore the rate of change of the curvature of the characteristics can produce intermodulation among the various channels to an objectionable degree. These problems present the necessity of highly linear amplifier characteristics with the requirements becoming ever more rigorous as the number of relay points increases. The linearity requirements may in many cases be the determining factor in fixing the applications of frequency division systems.

For the separation of individual channels after demodulation of the carrier and recovery of the complex modulating signal, band-pass filters are used. The band-pass characteristics of these filters, the frequency spacing between subcarriers, and the intelligence bandwidth determine the adequacy of the isolation of channels from each other. Conversely, the required frequency band occupied by the modulating signal will be determined by the filter characteristics, channel separation, and the intelligence bandwidths.

For FM wide-band operation recent investigations have shown that non-linearity of the phase characteristics of the tuned circuits between the modulator and the demodulator can be a source of objectionable intermodulation. This condition can be alleviated by broadening the tuned circuits sufficiently to provide satisfactory phase characteristics and by the use of phase corrective networks.

The minimum required bandwidths for operations of all of the frequency division systems are readily calculated to be

$$B_{min} = 2nf_w$$

The frequency separation between channels, f_w, will be determined largely by the ability to produce filters having the desired characteristics. With band-pass filters having infinitely steep sides and linear phase characteristics through the pass-band, f_w could be made twice the highest intelligence frequency of the channels (equal to the intelligence band for SS-XX systems). In practise, $f_w = k F_a$ where k is approximately 1.2 or somewhat greater for the SS-XX systems and approximately 2.2 or greater for the other systems.

3. Time Division Systems.

Since the modulating signal is composed of time-spaced samples of the several channels, system non-linearities

cannot produce intermodulation among the channels. The primary cause of the appearance in one channel output of undesired energy from other channels is the broadening out in time of the channel pulses so that they overlap. The distortion of a pulse passed through a tuned circuit or a filter is a function of the pulse shape and the filter characteristics; hence the undesired transfer of energy between adjacent pulses (ie, channels) is a function of pulse shape, pulse spacing, and RF band-pass characteristics of the system. It has been shown (Baeyer (2)) that the commonly used tuned circuit amplifiers have a response characteristic which approaches closely the shape of the probability function as the number of stages increases, and that the actual response probably lies between the idealized band-pass and the probability curve. If the Fourier integral is applied to the minimum length triangular pulses and the probability response function is assumed, the output pulse characteristics can be found.* The minimum length pulse will have a build-up and decay time and nominal duration τ which is the reciprocal of the bandwidth B. If the pulse repetition frequency is so chosen that the peak-to-peak pulse separation amounts to 2τ, the distortion of the pulses resulting from the less than infinite bandwidth gives the condition in which one pulse has not decayed completely to zero before the next pulse starts; and the amount of this overlapping is approximately 22 db on an

*Guilleman (6), Levy (10), Simmonds (20), Sullivan (22)

energy basis.* Note that the above pulse spacing corresponds to a separation at the base line of the undistorted pulses of an amount equal to τ. If the spacing at the bases is doubled, to 2τ, the interchannel energy is down to approximately 60db. This being a satisfactorily low value the bandwidths used have been determined on this basis. If the pulse separation at the base line is decreased (by width modulation, for instance), the bandwidth must be increased in the same proportion to maintain the same intermodulation ratio. The bandwidths required for the various systems are calculated on these requirements to be

$12nF_a$	for	PAM-AM			
$24nF_a$	for	PAM-FM	PWM-AM	PWM-FM	
$36nF_a$	for	PPM-AM	PPM-FM		
$63nF_a$	for	PCM-AM	PCM-FM	PNM-AM	PNM-FM

The question of cumulative distortions, though not generally as serious as in the frequency division systems, still exists in that progressive deterioration of the pulses through many repeater stations may set a practical limit to the number of such repeaters that can be used with reasonable bandwidth and receiver stability requirements. There is however one exception to this; namely the PCM systems.

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*Roberts and Simmonds (17)

When these systems are operated above the threshold level so that the maximum noise level prior to channel detection is below the channel clipping level, there is no way in which the noise can appear in the output circuit. This is readily seen when it is realized that detection of the intelligence is dependent only upon recognition of the presence or absence of a particular pulse and not upon its characteristics. This permits regeneration of a completely new undistorted set of pulses at each repeater station.

The comparison has actually been somwhat unfair to PCM in the matter of bandwidth. Unlike the other systems, in which relatively small overlapping of adjacent channel pulses can produce considerable inter-channel cross-talk, PCM can stand rather large overlapping without producing any cross-talk at all. Approximately optimum operation is such that with filters of the Gaussian characteristic, the bandwidth is adjusted so that the received pulses overlap at the 50% amplitude points; clipping and limiting at about 50% amplitude is used, followed by gating of the receiver at the midpoint of the pulse interval with a relatively narrow pulse supplied by local timing generators. With this method, a 96 channel system requires about 12mc RF bandwidth.*

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* Meacham (11)

CHAPTER VII

CONCLUSIONS

The PCM and the PNM systems have no near competitors when operated above the threshold. The PNM systems have somewhat less advantage than PCM; the figures in Table 1 do not fully indicate this unless it is realized that the pulses per code group were assumed the same in each system. As previously pointed out, under these conditions PCM is capable of distinguishing considerably more gradations of level in sampling the signal. If the two systems had been compared on the basis of equivalent numbers of sampling codes per group, PNM would have been found far inferior. It will be shown that the individual channel distortion is a function of the possible coding groups.

Pulse modulation systems in general are not quite as conservative of bandwidth as the frequency division systems but they provide somewhat better Merit Factors. The major difficulty associated with the frequency division systems, and the difficulty which makes the pulse systems (by their lack of it) so promising, is the requirement of extreme linearity of transmitters and receivers to avoid intermodulation. Further in favor of the pulse systems is the lack of the necessity for the relatively expensive frequency filtering equipment for channel separation. It

should be noted that in this connection the requirements of receiver and transmitter frequency stability will not be so great for the pulse systems. Receiver and transmitter circuits will in general be considerably less critical of adjustment for the pulse systems, although the circuitry may be somewhat more complex.

The great advantages offered by PCM warrant further consideration of this comparatively new system.

CHAPTER VIII

PCM SYSTEMS

1. Principles of operation.

It is possible by means of continuous variation of a pulse characteristic, such as width, heigth, or position, to transmit continuous variations of the intelligence waveform. However it has been shown that the presence of noise will also affect these pulse characteristics resulting in the superposition of an undesired noise on the desired intelligence output, and this effect is present in greater or less degree at all relative levels of the noise voltages and signal voltages. If a system could be devised which depended only upon the presence or absence of a pulse rather than the pulse characteristics, then as long as the maximum noise voltage was less than the voltage corresponding to the presence of a pulse, the presence or absence could always be distinguished without ambiguity due to the noise.

The transmission of continuous variations of the intelligence by such a system implies that the original continuous intelligence signal can be reconstructed from a series of on-off combinations. This would require an infinite number of pulses or pulse combinations, an obvious impracticality. Restricted to a finite number p of pulse combinations, one can transmit no more than p different

levels of intelligence information. The intelligence signal is therefore broken up into p quanta each of which is assigned one of the p possible combinations. At the receiving end the intelligence signal is synthetically approximated by the proper addition of the quanta corresponding to the received pulse combinations. This solution of the problem, first proposed by Reeves in a French patent in 1939, developed into PCM.

Since the quantizing process precludes the possibility of exact reconstruction of the intelligence signal, the amount of the consequent distortion should be determined. This has been found* to be given by

$$\%D = \frac{200}{p\sqrt{6}}$$

for the case of p quanta of equal size.

In practice, for speech or similarly complex signals the quantization process results in the introduction of a background similar to thermal noise**, the level of which is determined by the size of the quanta. To make the effects less annoying at low signal levels a form of pre-emphasis can be obtained by tapering the size of the quanta, the smaller sizes for the lower signal levels. See Figures 10(a) and 10(b). This taper can be obtained

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*Panter, Clavier and Grieg (14)
**Meacham (11)

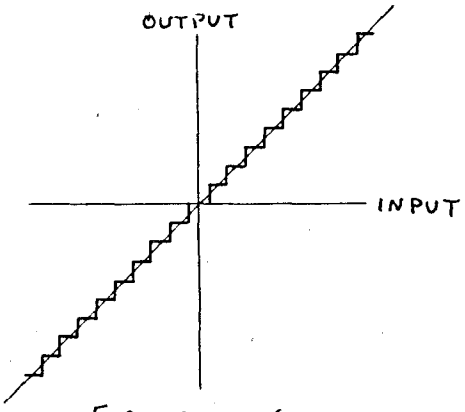


Figure 10 (a)

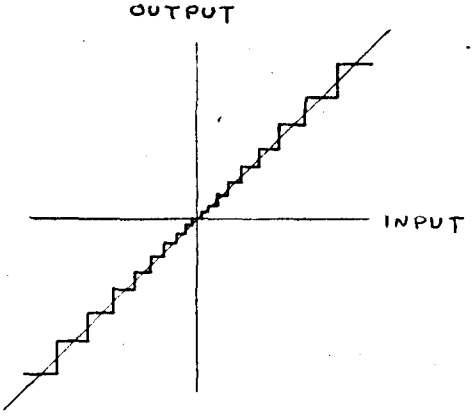


Figure 10 (b)

in the design of the coder mechanism. Alternatively, a linear (equal quanta) coder can be used and the intelligence signal subjected to amplitude compression prior to quantization; the result is a tapered step distribution with respect to the original uncompressed signal. The inverse procedure is used at the receiver to restore the amplitude scale of the initial intelligence.

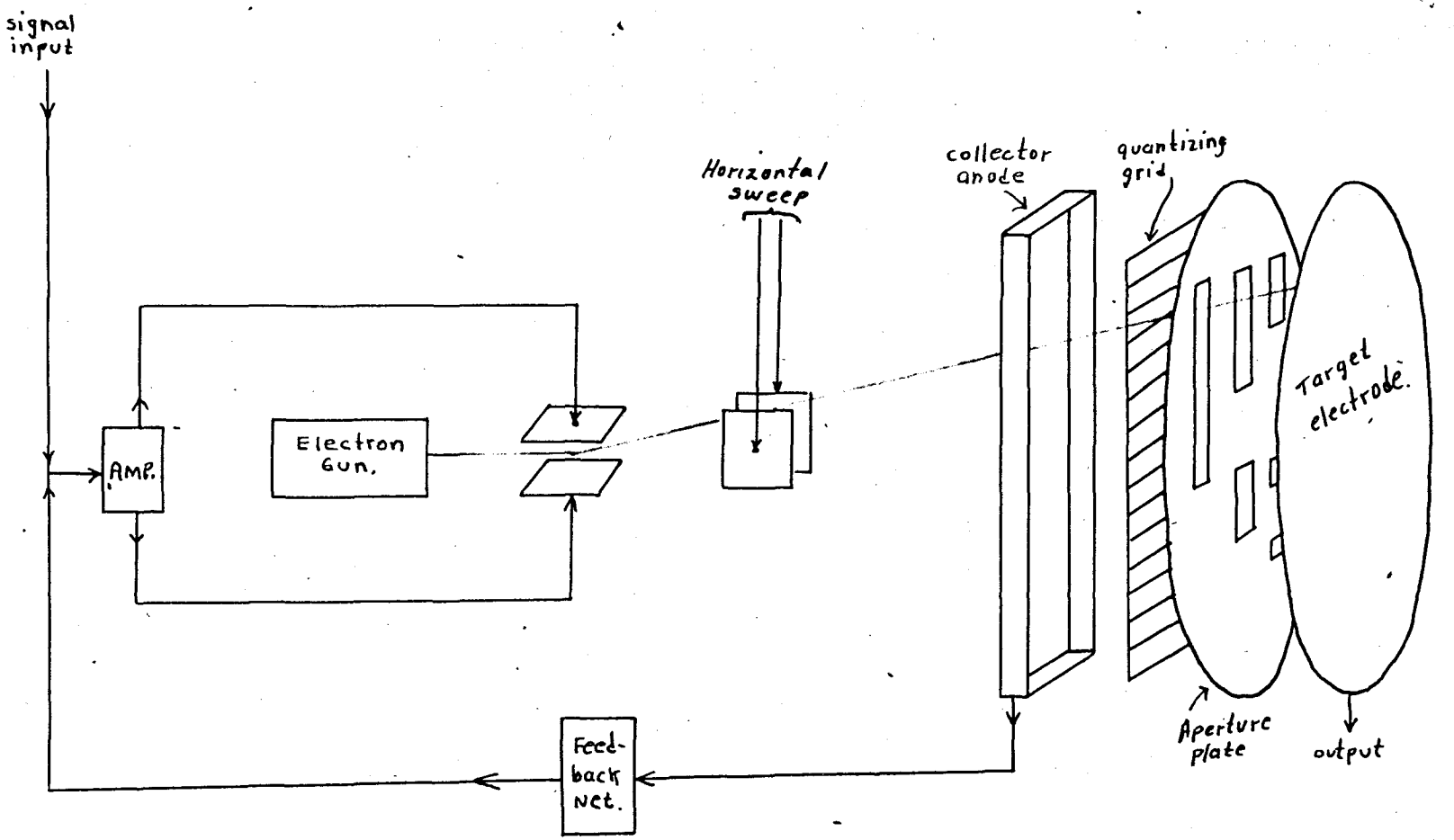
2. Coding.

The greatest single barrier to the development of a practical PCM communications system has been the lack of a suitable means of coding. The limitations commonly encountered were equipment complexity and lack of sufficient coding speed. The recent development by the Bell Laboratories* of an Electron Beam Deflection Coding Tube has removed this barrier.

A schematic diagram of this tube is shown in Fig.11, and its description and operation is as follows. The case for a three-digit code has been assumed for simplicity.

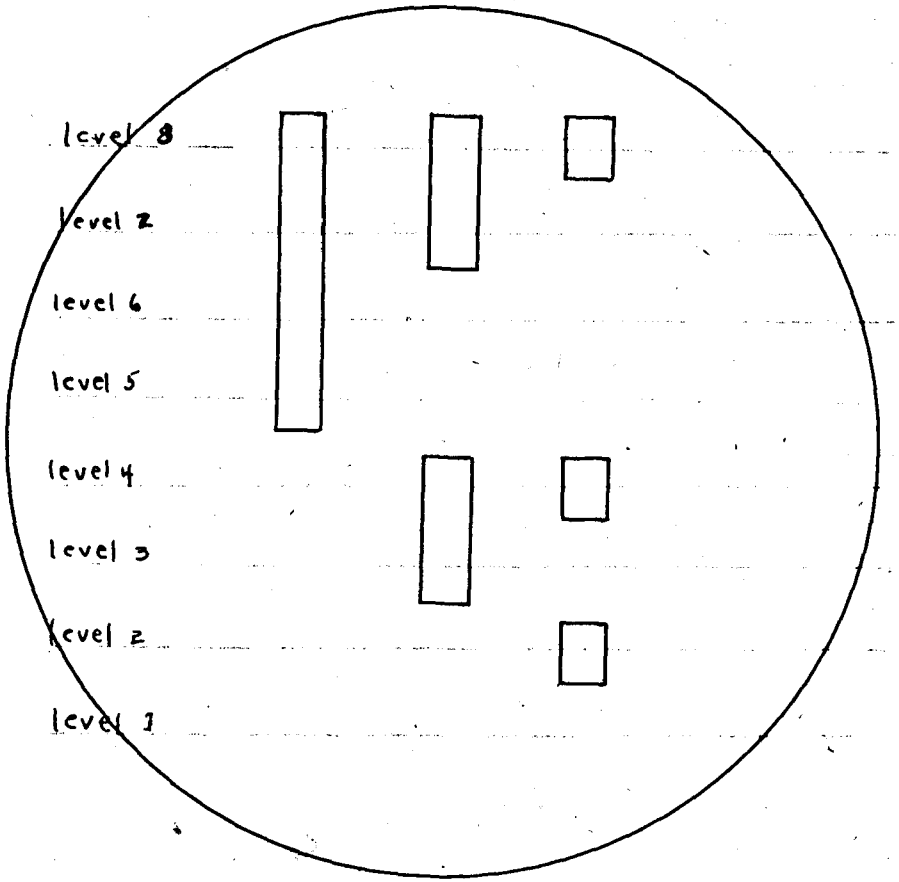
Coding is obtained by deflecting an electron beam horizontally across an aperture plate behind which is located the target electrode; an output signal is obtained through the target electrode circuit when the beam passes through an aperture in the aperture plate and strikes the target plate. The vertical positioning of the beam, which is constant during any one horizontal sweep, determines the number and relative positions of the apertures which the beam will strike during its horizontal travel. See Figure 12. The vertical position of the beam is determined by the amplitude of the intelligence signal at the moment of sampling. The intelligence input signal is quantized by the action of the quantizing grid, collector anode,

*Sears (18)

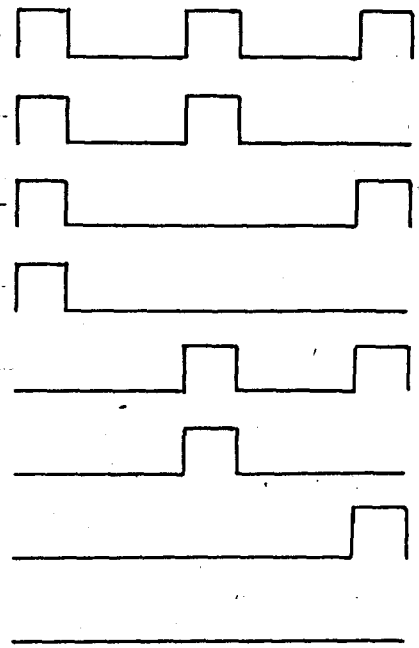


Schematic of Coding Tube Circuit.

Figure 11.



Aperture Plate



output signal

Three digit coding

Figure 12.

and an appropriate feed-back loop. The quantizing is located in front of the aperture plate as indicated in Figures 11 and 12. It will intercept a number of electrons proportional to the beam position: the quantizing grid current is maximum when the beam is directly on a grid wire and a minimum when the beam is just between two adjacent wires. A voltage derived from the quantizing grid current is applied through a feed-back amplifier to the vertical deflection plates; the net vertical deflection voltage is then the combination of the input signal voltage and the feed-back voltage. The beam is initially positioned vertically by the signal voltage alone. The feed-back loop is then energized and the beam takes a position determined by the sum of the signal and feed-back voltages. The amplitude and phase of the feed-back voltage is so adjusted that the equilibrium position taken by the beam is between two grid wires; perturbations of the signal voltage are balanced out by the feedback voltage such that they can cause only a slight vertical movement between the adjacent grid wires but cannot cause the beam to jump from one level to another. It has been found in practice that by processing the quantizing grid to give a secondary emission ratio of about 3, collecting the secondary electrons with a collector anode, and using the secondary electron current as the source of the feed-back voltage, that somewhat greater stability is obtainable.

In its final form, the tube utilized an aperture plate cut for a seven-digit code and produced pulses of about 0.8 micro-seconds duration with a total horizontal coding sweep time of about 10 micro-seconds.

3. Experimental system performance.

The Coding Tube described in the previous section has been used sucessfully in experimental equipment designed by the Bell Telephone Laboratories.* This system provided for 96 channel operation with an intelligence signal frequency band up to 3400 cps, operating at a carrier frequency of approximately 4000 mc with an RF bandwidth of about 12mc; as actually built and tested, only 12 channels were used. They were time-division multiplexed with a seven-digit code at an 8000 cps sampling rate. Two coders were used, one for the even-numbered channels and one for the odd-numbered channels to provide ample recovery time for the coders between successive input samples. Effective tapering of the quanta sizes was provided by amplitude compression prior to quantization.

The only significant source of noise was found to be the quantization, as previously discussed. The effects are indicated in Fig. 13. Overall cross-talk, a function only of proper design in the terminal equipment, was measured 66db down in the worst cases. With loud talkers into ten channels, measured cross-talk in the two idle channels averaged about one db above the quantizing noise.

The success of this early experimental work in closely

*Meacham (1)

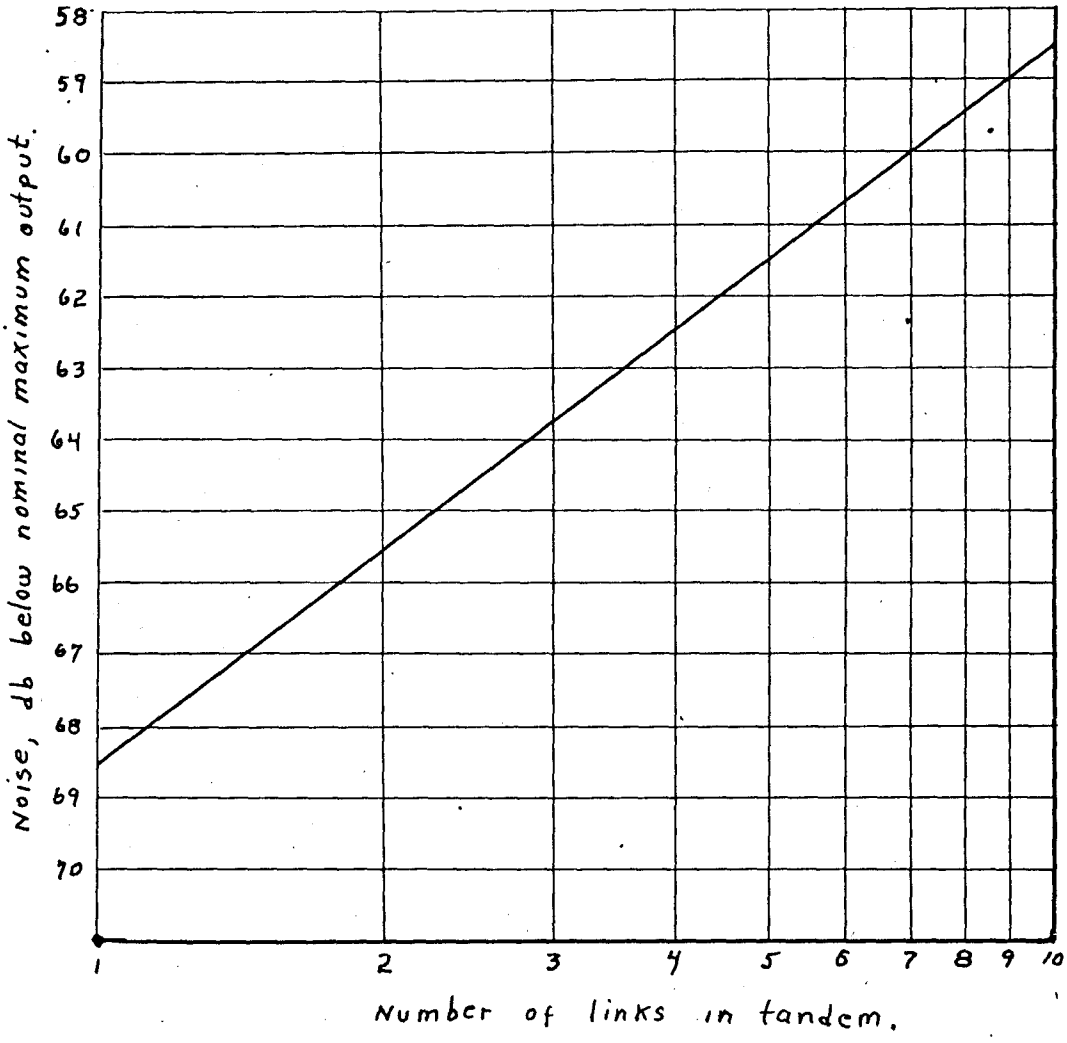

58
59
60
61
62
63
64
65
66
67
68
69
70
Noise, db below nominal maximum output.
1
2
3
4
5
6
7
8
9
10
Number of links in tandem.

Figure 13.

approaching theoretically predicted performances will undoubtedly result in continued effort to develop this highly promising method of modulation.

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